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CORRECTION OF CONTINUUM MECHANICS THEORETICAL PARADOXES (II)

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The monograph describes paradoxes of Stokes and Navier-Cauchy-Lame hypotheses. New equiations of viscous fluids dynamics, elasticity theory are theoretically justified. Current technologies of building effective numerical techniques for combined differential equation systems in true variables "velocity-pressure" are given in details.

The monograph is recommended for students, candidates for master's degree, post-graduate students, persons working for doctor's degree and researchers specialized in the sphere of fluid mechanics and elasticity theory.

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Chapter 8. FALLACY OF APPLYING STAGGERED GRIDS IN NUMERICAL CALCULATIONS

§1. Irreducible errors in schemes built upon staggered grids

We'll show *irreducible errors* on *Harlow-Welch* scheme. *Harlow-Welch* scheme /1/ uses in two-dimensional problems *three* staggered in relation to each other net domains, in three-dimensional problems - *four*. Let's limit ourselves to consideration of two-dimensional problem for *Navier* equations of incompressible fluid

$$\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uv}{\partial y}+\frac{1}{\rho}\frac{\partial p}{\partial x}=\nu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}), \tag{1}$$

$$\frac{\partial v}{\partial t}+\frac{\partial vu}{\partial x}+\frac{\partial v^2}{\partial y}+\frac{1}{\rho}\frac{\partial p}{\partial y}=\nu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}), \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

with initial $u\big|_{t=0}=d_u(x,y), v\big|_{t=0}=d_v(x,y),$

and boundary conditions $u\big|_S=\varphi_u(x,y,t), v\big|_S=\varphi_v(x,y,t)$.

In rectangular domain $\overline{\Omega}=[0\le x\le X,\quad 0\le y\le Y]$ with boundary S nodes of the major grid

$$\overline{\Omega}_h=\{x_i=ih_x, i=\overline{0,N_x}; y_j=jh_y, j=\overline{0,N_y}\}$$

are such that

$$\{x_0=0, x_{N_x}=X, j=\overline{0,N_y}\},\{y_0=0, y_{N_y}=Y, i=\overline{0,N_x}\}$$

are boundary nodes lying on physical boundary of the domain. Two additional grids $\overline{\Omega}_u$, $\overline{\Omega}_v$ are staggered relative to this main grid:

$$\overline{\Omega}_u=\{x_{i+\frac{1}{2}}=(i+\frac{1}{2})h_x, i=\overline{-1,N_x}; y_j=jh_y, j=\overline{0,N_y}\},$$

$$\overline{\Omega}_v=\{x_i=ih_x, i=\overline{0,N_x}; y_{j+\frac{1}{2}}=(j+\frac{1}{2})h_y, j=\overline{-1,N_y}\},$$

Sets of internal nodes of these three grids are denoted accordingly

$$\Omega_h = \{x_i = ih_x, i = \overline{1, N_x - 1}; y_j = jh_y, \ j = \overline{1, N_y - 1}\},$$

$$\Omega_u = \{x_{i+\frac{1}{2}} = (i + \frac{1}{2})h_x, i = \overline{0, N_x - 1}; y_j = jh_y, \ j = \overline{1, N_y - 1}\},$$

$$\Omega_v = \{x_i = ih_x, i = \overline{1, N_x - 1}; y_{j+\frac{1}{2}} = (j + \frac{1}{2})h_y, \ j = \overline{0, N_y - 1}\},$$

Time grid $\overline{\Omega}_\tau = \{t_n = n\tau, \quad n = 0,1,\ldots,N_\tau\}$.

Used are index notations of grid functions

$$f_{ij}^n = f(x_i, y_j, t_n), f_{i\pm\frac{1}{2}j}^n = f(x_{i\pm\frac{1}{2}}, y_j, t_n),$$

$$f_{ij\pm\frac{1}{2}}^n = f(x_i, y_{j\pm\frac{1}{2}}, t_n), f_{i\pm\frac{1}{2}j\pm\frac{1}{2}}^n = f(x_{i\pm\frac{1}{2}}, y_{j\pm\frac{1}{2}}, t_n),$$

$$f_{ij}^{n+1} = f(x_i, y_j, t_{n+1})$$

The *Harlow-Welch* scheme uses three grids, its own grid domain is built for each equation. Therefore equation (1) approximated in nodes of grid Ω_u:

$$\frac{u_{i+\frac{1}{2}j}^{n+1} - u_{i+\frac{1}{2}j}^n}{\tau} + \frac{u_{i+1j}^{n2} - u_{ij}^{n2}}{h_x} + \frac{u_{i+\frac{1}{2}j+\frac{1}{2}}^n v_{i+\frac{1}{2}j+\frac{1}{2}}^n - u_{i+\frac{1}{2}j-\frac{1}{2}}^n v_{i+\frac{1}{2}j-\frac{1}{2}}^n}{h_y} +$$

$$+ \frac{p_{i+1j}^n - p_{ij}^n}{\rho h_x} = \nu\left(\frac{u_{i+\frac{3}{2}j}^n - 2u_{i+\frac{1}{2}j}^n + u_{i-\frac{1}{2}j}^n}{h_x^2} + \frac{u_{i+\frac{1}{2}j+1}^n - 2u_{i+\frac{1}{2}j}^n + u_{i+\frac{1}{2}j-1}^n}{h_y^2}\right),$$

$$i = 0,\ldots,N_x - 1, \quad j = 1,\ldots,N_y - 1, \qquad (4)$$

equation (2) is approximated in nodes of grid Ω_v:

$$\frac{v_{ij+\frac{1}{2}}^{n+1} - v_{ij+\frac{1}{2}}^n}{\tau} + \frac{u_{i+\frac{1}{2}j+\frac{1}{2}}^n v_{i+\frac{1}{2}j+\frac{1}{2}}^n - u_{i-\frac{1}{2}j+\frac{1}{2}}^n v_{i-\frac{1}{2}j+\frac{1}{2}}^n}{h_x} + \frac{v_{ij+1}^{n2} - v_{ij}^{n2}}{h_y} +$$

$$+\frac{p_{ij+1}^n - p_{ij}^n}{\rho h_y} = \nu\left(\frac{v^n_{i+1j+\frac{1}{2}} - 2v^n_{ij+\frac{1}{2}} + v^n_{i-1j+\frac{1}{2}}}{h_x^2} + \frac{v^n_{ij+\frac{3}{2}} - 2v^n_{ij+\frac{1}{2}} + v^n_{ij-\frac{1}{2}}}{h_y^2}\right),$$

$$i = 1,\ldots,N_x - 1, \quad j = 0,\ldots,N_y - 1 , \qquad (5)$$

continuity equation (3) is approximated in nodes of the main grid $\overline{\Omega}_h$:

$$\frac{u^{n+1}_{i+\frac{1}{2}j} - u^{n+1}_{i-\frac{1}{2}j}}{h_x} + \frac{v^{n+1}_{ij+\frac{1}{2}} - v^{n+1}_{ij-\frac{1}{2}}}{h_y} = 0 \ , \overline{i = 0, N_x}, j = \overline{0, N_y} \qquad (6)$$

With values $i = 0, j = 1,\ldots,N_y - 1,$ (6) includes $N_y - 1$ values $u^{n+1}_{-\frac{1}{2}j}$ and when $i = N_x$, $j = 1,\ldots,N_y - 1$ (6) includes $N_y - 1$ of values $u^{n+1}_{N_x+\frac{1}{2}j}$ of unknown functions beyond physical domain $\overline{\Omega}$. Similarly, when values $j = 0, i = 1,\ldots,N_x - 1$, (6) includes $N_x - 1$ of values $v_{i-\frac{1}{2}N_y}$ and when $j = N_y$, $i = 1,\ldots,N_x - 1$, (6) includes $N_x - 1$ of values $v^{n+1}_{iN_y+\frac{1}{2}}$ of unknown functions beyond physical domain $\overline{\Omega}$. Along with this unknown $u^n_{-\frac{1}{2}j}$ are included into (4) when $i = 0, j = 1,\ldots,N_y - 1,$ and $u^{n+1}_{N_x+\frac{1}{2}j}$ are included into (4) when $i = N_x$-1, $j = 1,\ldots,N_y - 1$.

Here arises a natural question: how to define them? Equations (6) are not suitable for this purpose. According to the general concept, these equations are necessary for computation of pressure

$$p_{ij}^{n}\ , i=0,\ldots,N_x\,,\qquad j=0,\ldots,N_y$$

(nodes on rectangle vertices are not included in here).

From equations (4) and (5) singled out are

$$u_{i+\frac{1}{2}j}^{n+1}=-\tau\frac{p_{i+1j}^{n}-p_{ij}^{n}}{\rho h_x}+R_{i+\frac{1}{2}j}^{n}\ , i=0,\ldots,N_x-1,\ j=1,\ldots,N_y-1\,,\qquad (7)$$

$$v_{ij+\frac{1}{2}}^{n+1}=-\tau\frac{p_{ij+1}^{n}-p_{ij}^{n}}{\rho h_y}+Q_{ij+\frac{1}{2}}^{n}\ , i=1,\ldots,N_x-1,\ j=0,\ldots,N_y-1\qquad (8)$$

and are substituted into continuity equation (6) in nodes with numbers $i=1,\ldots,N_x-1,\quad j=1,\ldots,N_y-1$. In the result obtained is a system of difference equations for pressure

$$\frac{p_{i+1j}^{n}-2p_{ij}^{n}+p_{i-1j}^{n}}{\rho h_x^2}+\frac{p_{ij+1}^{n}-2p_{ij}^{n}+p_{ij-1}^{n}}{\rho h_y^2}=$$

$$=\frac{R_{i+\frac{1}{2}j}^{n}-R_{i-\frac{1}{2}j}^{n}}{\tau h_x}+\frac{Q_{ij+\frac{1}{2}}^{n}-Q_{ij-\frac{1}{2}}^{n}}{\tau h_y}\ ,\qquad (9)$$

$$i=1,\ldots,N_x-1,\qquad j=1,\ldots,N_y-1$$

Boundary conditions for pressure are derived from the same equations (6), but considered in boundary nodes

$$i=0,\ j=1,\ldots,N_y-1;\, i=0,\ j=1,\ldots,N_y-1;$$

$$j=0, i=1,\ldots,N_x-1;\ j=N_y\ , i=1,\ldots,N_x-1$$

Let’s limit to boundary nodes $i=0, j=1,\ldots,N_y-1$, to show arisen problem of defining $u_{-\frac{1}{2}j}^{n+1}$.

Substituting when $i=0,\ j=1,\ldots,N_y-1$ into continuity equation (6) values (7), we obtain differential boundary conditions for pressure

$$\frac{(-\tau\dfrac{p_{1j}^{n}-p_{0j}^{n}}{\rho h_x}+R_{\frac{1}{2}j}^{n})-u_{-\frac{1}{2}j}^{n+1}}{h_x}+\frac{v_{0j+\frac{1}{2}}^{n+1}-v_{0j-\frac{1}{2}}^{n+1}}{h_y}=0\ ,\ j=\overline{1,N_y-1},\qquad (10)$$

where due to boundary conditions for velocity components it is provided

$$v_{0j\pm\frac{1}{2}}^{n+1}=\varphi_v(x_0,y_{j\pm\frac{1}{2}},t),\quad j=1,\ldots,N_y-1$$

As can be seen, boundary condition (10) includes unknown $u_{-\frac{1}{2}j}^{n+1}$, $j=1,\ldots,N_y-1$. Similar will be entry of unknown quantities into boundary conditions for pressure

$$u_{N_x+\frac{1}{2}j}^{n+1},\ j=1,\ldots,N_y-1;\ \ v_{i-\frac{1}{2}}^{n+1}\ ,\ \ v_{iN_y+\frac{1}{2}}^{n+1}\ ,i=1,\ldots,N_x-1;$$

§2. On additional boundary conditions and on the problem of one-valued solvability

So, for defining unknown quantities

$$u_{-\frac{1}{2}j}^{n+1},u_{N_x+\frac{1}{2}j}^{n+1},\ j=1,\ldots,N_y-1,v_{i-\frac{1}{2}}^{n+1},v_{iN_y+\frac{1}{2}}^{n+1},i=1,\ldots,N_x-1$$

required are *additional boundary conditions* for calculations, which contradicts the initial setting of initially-boundary problem for equations (1),(2),(3), because there are enough available boundary conditions $u\big|_S=\varphi_u(x,y,t),v\big|_S=\varphi_v(x,y,t)$.

For example, *Poisson* equation with boundary condition of *Dirichlet*

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=f(x,y)\ ,\ \Phi\big|_S=\varphi$$

has a unique solution, while in applying *staggered grids* for defining

$$\Phi_{-\frac{1}{2}j}^{n+1},\Phi_{N_x+\frac{1}{2}j}^{n+1},\ j=1,\ldots,N_y-1,\qquad \Phi_{i-\frac{1}{2}}^{n+1},\Phi_{iN_y+\frac{1}{2}}^{n+1},i=1,\ldots,N_x-1,$$

Required is *additionally boundary condition* of $\Lambda\Phi\big|_S=\psi$ type, where, furthermore, arises a problem of defining type of differential

operator Λ and function $\psi = \psi(x, y)$ on boundary of domain S. And right away arises a question: is there *such unique solution of Poisson* equation with two boundary conditions $\Phi\big|_s = \varphi$ and $\Lambda\Phi\big|_S = \psi$, which would match with solution of *Poisson* equation with one boundary condition $\Phi\big|_s = \varphi$?

Literature

1. Harlow F.H., Welch J.E. Numerical calculation of timedependent viscous incompressible flow of fluid with freesurface // Phys. of Fluids, 1965, v.8, No.12, p.2182-2189.
2. Лилли Д. (D.G.Lilley) // Простой метод расчета скоростей и давления в сильно завихренных течениях.- Ракетная техника и космонавтика.1976, т.14, июнь, с.57-67.
3. Белоцерковский О.М. Численные методы механики сплошной среды. - М.,Физматгиз, 1985.

Chapter 9. TECHNOLOGIES OF EFFECTIVE SCHEMES BUILDING ON A SINGLE GRID

Families of schemes, *using only one main grid* $\overline{\Omega}_h$ for equations *Navier-Stokes* were developed and theoretically justified in monograph of the author /1/.

Let's stop only on some of them on example of a two-dimensional problem. Calculation of disk longitudinal flow, results of which confirmed fallacy of *Prandtl* equations, was carried out with the use of one of these schemes.

§1. Formulation of the problem longitudinal flow past a plate

For calculation of two-dimensional flow past a plate, let's use closed system of equations from **Chapter 3**:

$$\rho[\frac{\partial\bar{v}_i}{\partial t}+\sum_j \bar{v}_j\frac{\partial\bar{v}_i}{\partial x_j}+\sum_j\frac{\partial v'_j v'_i}{\partial x_j}-\frac{t^o}{2}\sum_j\frac{\partial^2 v'_j v'_i}{\partial t\partial x_j}]+\frac{\partial\bar{p}}{\partial x_i}=$$

$$=\rho\bar{F}_i+\mu\Delta\bar{v}_i, \qquad \boldsymbol{i=1,2,3}, \tag{1}$$

$$\sum_i\frac{\partial\bar{v}_i}{\partial x_i}=0, \tag{2}$$

$$\rho[\frac{\partial v'_i}{\partial t}+\sum_j(\bar{v}_j\frac{\partial v'_i}{\partial x_j}+v'_j\frac{\partial\bar{v}_i}{\partial x_j})+\frac{t^o}{2}\sum_j\frac{\partial^2 v'_j v'_i}{\partial t\partial x_j}]+\frac{\partial p'}{\partial x_i}=$$

$$=\rho F'_i+\mu\Delta v'_i, \qquad \boldsymbol{i=1,2,} \tag{3}$$

$$\sum_i\frac{\partial v'_i}{\partial x_i}=0, \tag{4}$$

$$\bar{v}_i\Big|_\sigma=\bar{\varphi}_i, \bar{v}_i\Big|_{t=0}=\bar{v}_i^0, v'_i\Big|_\sigma=\varphi'_i, v'_i\Big|_{t=0}=v'^0_i \tag{5}$$

for simulating of both laminar ($v'_i\equiv 0, i=1,2,3$) as well as turbulent ($v'_i\neq 0, i=1,2,3$) modes (interleaved mode is accounted for automatically) of flow past a plate and other objects. In notations $\bar{v}_1\equiv u, \bar{v}_2\equiv v, \bar{p}\equiv p, \bar{F}_1\equiv F_x, \bar{F}_2\equiv F_y, v'_1\equiv u', v'_2\equiv v'$ system will take the form of

$$\rho\{\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial u'^2}{\partial x}+\frac{\partial u'v'}{\partial y}-\frac{t^o}{2}\frac{\partial}{\partial t}(\frac{\partial u'^2}{\partial x}+\frac{\partial u'v'}{\partial y})\}+\frac{\partial p}{\partial x}=$$

$$=\rho F_x+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}), \tag{6}$$

$$\rho\{\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{\partial u'v'}{\partial x}+\frac{\partial v'^2}{\partial y}-\frac{t^o}{2}\frac{\partial}{\partial t}(\frac{\partial u'v'}{\partial x}+\frac{\partial v'^2}{\partial y})\}+\frac{\partial p}{\partial y}=$$

$$=\rho F_y+\mu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}), \tag{7}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{8}$$

$$\rho\{\frac{\partial u'}{\partial t}+u\frac{\partial u'}{\partial x}+v\frac{\partial u'}{\partial y}+u'\frac{\partial u}{\partial x}+v'\frac{\partial u}{\partial y}+\frac{t^o}{2}\frac{\partial}{\partial t}(\frac{\partial u'^2}{\partial x}+\frac{\partial u'v'}{\partial y})\}+\frac{\partial p'}{\partial x}=$$

$$=\rho F_x^{'}+\mu(\frac{\partial^2 u'}{\partial x^2}+\frac{\partial^2 u'}{\partial y^2}), \tag{9}$$

$$\rho\{\frac{\partial v'}{\partial t}+u\frac{\partial v'}{\partial x}+v\frac{\partial v'}{\partial y}+u'\frac{\partial v}{\partial x}+v'\frac{\partial v}{\partial y}+\frac{t^o}{2}\frac{\partial}{\partial t}(\frac{\partial u'v'}{\partial x}+\frac{\partial v'^2}{\partial y})]]\}+\frac{\partial p'}{\partial y}=$$

$$=\rho F_y^{'}+\mu(\frac{\partial^2 v'}{\partial x^2}+\frac{\partial^2 v'}{\partial y^2}), \tag{10}$$

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}=0, \tag{11}$$

Equations (6),(7),(8) transform into *Navier* incompressible fluids equations when $u'\equiv 0, v'\equiv 0, \forall(x,y,t)$.

At the zero time $t=0$, fluid is in state of rest in laminar flow:

$$u(x,y,0)=0,\ v(x,y,0)=0, 0\le x\le L+2l, 0\le y\le H,$$

at some moment of time $t^*=t_{n^*}$ by flow obtained for this moment of time by numerical method, *imposed is* a “random” disturbance, i.e. in the grid nodes at this moment are assumed

$$u_{ij}^{n^*}=u_{ij}^{n}+\varsigma_{ij}, v_{ij}^{n^*}=v_{ij}^{n}+\xi_{ij}, i=1,.., N_x-1, j=1,..., N_y-1,$$

where $\varsigma_{ij}=\varsigma(x_i,y_j)$ and $\xi_{ij}=\xi(x_i,y_j)$ close to zero random functions, simulating initial perturbation in a flow (they can be set with the help of random numbers generator).

Boundary conditions: incident flow has the following velocities

$$x=0, u(0,y,t)=U_\infty\exp(-b'/t), v(0,y,t)=0,$$

$$u'(0,y,t)=\zeta(y,t)-\text{random function},\quad v'(0,y,t)=0, \tag{12}$$

$$0\le y\le H, b'=const>0,\quad |\zeta(y,t)|<<U_\infty\ \ eet.,$$

on the plate – condition of adhesion and impermeability

$$y=0,\ u(x,0,t)=0, u'(x,0,t)=0$$
$$v(x,0,t)=0, v'(x,0,t)=0, l \le x \le L+l, \qquad (13)$$

at the flow outlet “soft boundary conditions” are set

$$x=L+2l, \frac{\partial^2 u(L+2l,y,t)}{\partial x^2}=0, \frac{\partial^2 u'(L+2l,y,t)}{\partial x^2}=0,$$
$$\frac{\partial^2 v(L+2l,y,t)}{\partial x^2}=0, \frac{\partial^2 v'(L+2l,y,t)}{\partial x^2}=0, 0 \le y \le H, \qquad (14)$$

in the upper part of flow, extremum conditions

$$y=H,\ \frac{\partial u(x,H,t)}{\partial y}=0, \frac{\partial u'(x,H,t)}{\partial y}=0,$$
$$\frac{\partial v(x,H,t)}{\partial y}=0, \frac{\partial v'(x,H,t)}{\partial y}=0, 0 \le x \le L+2l \qquad (15)$$

Before beginning of the plate and after the plate symmetric flow-around conditions are set

$$y=0, \frac{\partial u(x,0,t)}{\partial y}=0, v(x,0,t)=0, \frac{\partial u'(x,0,t)}{\partial y}=0, v'(x,0,t)=0,$$
$$0 < x < l, L+l < x \le L+2l$$

Transition to dimensionless variables

For transition to dimensionless variables selected as scale values are: for velocities U_∞, for linear dimensions – plate length L, for time L/U_∞, for pressure ρU_∞^2. In dimensionless quantities,

$$\hat{t}=tU_\infty/L, \hat{x}=x/L,$$
$$\hat{y}=y/L, \hat{u}=u/U_\infty, \hat{v}=v/U_\infty, \hat{p}=p/(\rho U_\infty^2), Dg=t^o U_\infty/L,$$
$$\hat{F}_x=F_x/g. \hat{F}_y=F_y/g. Fr=U_\infty^2/(gL)-Froude \quad number,$$

$Re=\rho U_\infty L/\mu$ system (6),(7),(8) (signs are omitted) has the form:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial u'^2}{\partial x}+\frac{\partial u'v'}{\partial y}-\frac{Dg}{2}\frac{\partial}{\partial t}\left(\frac{\partial u'^2}{\partial x}+\frac{\partial u'v'}{\partial y}\right)+\frac{\partial p}{\partial x}=$$

$$=\frac{F_x}{Fr}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}), \tag{16}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{\partial u'v'}{\partial x}+\frac{\partial v'^2}{\partial y}-\frac{Dg}{2}\frac{\partial}{\partial t}(\frac{\partial u'v'}{\partial x}+\frac{\partial v'^2}{\partial y})+\frac{\partial p}{\partial y}=$$

$$=\frac{F_y}{Fr}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}), \tag{17}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{18}$$

Equations for disturbances (pulsations) (9),(10),(11) are also transformed into similar dimensionless form:

$$\frac{\partial u'}{\partial t}+u\frac{\partial u'}{\partial x}+v\frac{\partial u'}{\partial y}+u'\frac{\partial u}{\partial x}+v'\frac{\partial u}{\partial y}+\frac{Dg}{2}\frac{\partial}{\partial t}(\frac{\partial u'^2}{\partial x}+\frac{\partial u'v'}{\partial y})+\frac{\partial p'}{\partial x}=$$

$$=\frac{F_x'}{Fr}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 u'}{\partial x^2}+\frac{\partial^2 u'}{\partial y^2}), \tag{19}$$

$$\frac{\partial v'}{\partial t}+u\frac{\partial v'}{\partial x}+v\frac{\partial v'}{\partial y}+u'\frac{\partial v}{\partial x}+v'\frac{\partial v}{\partial y}+\frac{Dg}{2}\frac{\partial}{\partial t}(\frac{\partial u'v'}{\partial x}+\frac{\partial v'^2}{\partial y})+\frac{\partial p'}{\partial y}=$$

$$=\frac{F_y'}{Fr}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 v'}{\partial x^2}+\frac{\partial^2 v'}{\partial y^2}), \tag{20}$$

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}=0, \tag{21}$$

Boundary conditions (12)-(15)

$$x=0, u(0,y,t)=\exp(-b^o/t),$$

$$v(0,y,t)=0, 0\le y\le H/L, b^o=const>0;$$

$$x=1+2l/L, \frac{\partial^2 u(1+2l/L,y,t)}{\partial x^2}=0,$$

$$\frac{\partial^2 v(1+2l/L,y,t)}{\partial x^2}=0, 0\le y\le H/L;$$

$$y=0,\ u(x,0,t)=0,\ v(x,0,t)=0, l/L \le x \le 1+l/L;$$

$$y=H/L, \frac{\partial u(x,H/L,t)}{\partial y}=0,$$

$$\frac{\partial v(x,H/L,t)}{\partial y}=0, 0\le x \le 1+2l/L;$$

$$y=0, \frac{\partial u(x,0,t)}{\partial y}=0,$$

$$v(x,0,t)=0, 0<x<l/L, 1+l/L<x\le 1+2l/L$$

Similar dimensionless boundary conditions occur for pulsation equations as well.

Initially-boundary problem for (16)-(21) is solved theoretically by justified difference methods *Jakupov K.B.* /1/), detailed technique of which is given below.

§2. Technologies of building difference schemes in vicious fluid dynamic equations on a single grid. Semi-implicit difference scheme

For stating principles of building and algorithm of realizing difference schemes for vicious fluid dynamics equations on a single grid, the most suitable is the family of parametric semi-implicit schemes from /1/($\overline{\alpha}=0, \overline{\beta}=1$), which is represented by example of equations (16), (17), (18). In dimensionless domain $0\le x\le 1+2l/L$, $0\le y\le H/L$ grid $\overline{\Omega}_h=\{x_i, i=0,\dots,N_x; y_j, j=0,\dots,N_y\}$ is introduced, and time grid $\Omega_{\tau 0}=\{t_n, n=0,1,\dots,N_\tau\}$ with pitches

$$h_{xi}=x_i-x_{i-1}>0, i=1,\dots,N_x, h_{yj}=y_j-y_{j-1}>0, j=1,\dots,N_y,$$

$$x_0=0, y_0=0, x_{N_x}=1+2l/L, y_{N_y}=H/L, \tau_n=t_n-t_{n-1}>0, n=1,\dots,N_\tau$$

Grid pitches near streamlined surface must be of the following order $h\approx 1/\sqrt{\mathrm{Re}}$. Monotonic scheme **Chapter 6** approximating convection members with 2nd order of accuracy is applied (see

Jakupov K.B./2/). Scheme is semi-implicit in time because pressure and continuity equation are taken on the upper time layer:

$$\frac{u_{ij}^{n+1}-u_{ij}^{n}}{\tau_{n+1}}+\frac{|u_{ij}^{n}|+u_{ij}^{n}}{2}u_{\bar{x}}^{n}+\frac{u_{ij}^{n}-|u_{ij}^{n}|}{2}u_{x}^{n}+\frac{|v_{ij}^{n}|+v_{ij}^{n}}{2}u_{\bar{y}}^{n}+\frac{v_{ij}^{n}-|v_{ij}^{n}|}{2}u_{y}^{n}+$$

$$+\{(u'^2)_{\tilde{x}}^{n}+(u'v')_{\tilde{y}}^{n}-\frac{Dg}{2}[(u'^2)_{\tilde{x}}^{n}+(u'v')_{\tilde{y}}^{n}-(u'^2)_{\tilde{x}}^{n-1}-(u'v')_{\tilde{y}}^{n-1}]/\tau_n\}+$$

$$+\frac{p_{i+1j}^{n+1}-p_{ij}^{n+1}}{h_{xi+1}}=\frac{F_x}{Fr}+\frac{2\mu_{uxv}^{v}}{h_{xi+1}+h_{xi}}(u_x^n-u_{\bar{x}}^n)+\frac{2\mu_{vyu}^{u}}{h_{yj+1}+h_{yj}}(u_y^n-u_{\bar{y}}^n)],(22)$$

$$\frac{v_{ij}^{n+1}-v_{ij}^{n}}{\tau_{n+1}}+\frac{|u_{ij}^{n}|+u_{ij}^{n}}{2}v_{\bar{x}}^{n}+\frac{u_{ij}^{n}-|u_{ij}^{n}|}{2}v_{x}^{n}+\frac{|v_{ij}^{n}|+v_{ij}^{n}}{2}v_{\bar{y}}^{n}+\frac{v_{ij}^{n}-|v_{ij}^{n}|}{2}v_{y}^{n}+$$

$$+\{(u'v')_{\tilde{x}}^{n}+(v'^2)_{\tilde{y}}^{n}-\frac{Dg}{2}[(u'v')_{\tilde{x}}^{n}+(v'^2)_{\tilde{y}}^{n}-(u'v')_{\tilde{x}}^{n-1}-(v'^2)_{\tilde{y}}^{n-1}]/\tau_n\}+$$

$$+\frac{p_{ij+1}^{n+1}-p_{ij}^{n+1}}{h_{yj+1}}=\frac{F_y}{Fr}+\frac{2\mu_{uxv}^{v}}{h_{xi+1}+h_{xi}}(v_x^n-v_{\bar{x}}^n)+\frac{2\mu_{vyu}^{u}}{h_{yj+1}+h_{yj}}(v_y^n-v_{\bar{y}}^n)],\quad (23)$$

$$i=1,\ldots,N_x-1,\ j=1,\ldots,N_y-1;$$

$$\frac{u_{ij}^{n+1}-u_{i-1j}^{n+1}}{h_{xi}}+\frac{v_{ij}^{n+1}-v_{ij-1}^{n+1}}{h_{yj}}=0,\quad i=1,\ldots,N_x,\ j=1,\ldots,N_y;\ \ n=\overline{0,N_\tau}\ ,\quad (24)$$

where coefficients in dissipative members are equal

$$\Re=1+\mathrm{Re}(\frac{|u_{ij}^{n}|+u_{ij}^{n}}{4}h_{xi}++\frac{|u_{ij}^{n}|-u_{ij}^{n}}{4}h_{xi+1}+\frac{|v_{ij}^{n}|+v_{ij}^{n}}{4}h_{yj}+\frac{|v_{ij}^{n}|-v_{ij}^{n}}{4}h_{yj+1}),$$

$$\mu_{uxv}^{v}=(\frac{1}{\mathrm{Re}}+\frac{|v_{ij}^{n}|+v_{ij}^{n}}{4}h_{yj}+\frac{|v_{ij}^{n}|-v_{ij}^{n}}{4}h_{yj+1})/\Re,$$

$$\mu_{vyu}^{u}=(\frac{1}{\mathrm{Re}}+\frac{|u_{ij}^{n}|+u_{ij}^{n}}{4}h_{xi}+\frac{|u_{ij}^{n}|-u_{ij}^{n}}{4}h_{xi+1})/\Re$$

On uniform mesh $h_{yj+1} = h_{yj} = h_y, \forall j, h_{xi+1} = h_{xi} = h_x, \forall i$ these coefficients are reduced

$$\mu_{uxv}^{v} = [\frac{1}{\mathrm{Re}} + \frac{|v_{ij}^{n}| h_y}{2}]/[1 + \mathrm{Re}(\frac{|u_{ij}^{n}| h_x}{2} + \frac{|v_{ij}^{n}| h_y}{2})],$$

$$\mu_{vyu}^{u} = [\frac{1}{\mathrm{Re}} + \frac{|u_{ij}^{n}| h_x}{2}]/[1 + \mathrm{Re}(\frac{|u_{ij}^{n}| h_x}{2} + \frac{|v_{ij}^{n}| h_y}{2})]$$

With values

$$\mu_{uxv}^{v} = \frac{1}{\mathrm{Re}},\ \mu_{vyu}^{u} = \frac{1}{\mathrm{Re}}$$

the scheme has the 1st order of accuracy.

Initial conditions are set on grid

$$\Omega_h = \{x_i, i = 1,...,N_x - 1; y_j, j = 1,...,N_y - 1\}:$$

$$u_{ij}^{0} = 0, v_{ij}^{0} = 0, i = \overline{1, N_x - 1}, j = \overline{1, N_y - 1} \tag{25}$$

Boundary conditions (18)-(21) are set in boundary nodes (the below formulae are given on uniform grid):

$$S_h = \{i = 0, i = N_x, j = \overline{0, N_y}; j = 0, j = N_y, i = \overline{0, N_x}\}, \overline{\Omega}_h = \Omega_h \bigcup S_h;$$

$$x = 0, u_{0j}^{n+1} = e^{-\frac{b^o}{t_{n+1}}}, v_{0j}^{n+1} = 0, j = \overline{0, N_y - 1}, \tag{26}$$

$$\begin{gathered} x = 1 + 2l/L, u_{N_x j}^{n+1} = 2u_{N_x - 1j}^{n+1} - u_{N_x - 2j}^{n+1}, \\ v_{N_x j}^{n+1} = 2v_{N_x - 1j}^{n+1} - v_{N_x - 2j}^{n+1}, 0 \le j \le N_y; \end{gathered} \tag{27}$$

$$on \quad plate \quad u_{i0}^{n+1} = 0, v_{i0}^{n+1} = 0, mk \le i \le mkk, \tag{28}$$

before the beginning and after the end of the plate пластины

$$y = 0, u_{i0}^{n+1} = (4u_{i1}^{n+1} - u_{i2}^{n+1})/3, v_{i0}^{n+1} = 0, 1 \le i \le mk - 1, mkk + 1 \le i \le N_x,$$

$$\begin{gathered} y = \frac{H}{L}: u_{iN_y}^{n+1} = (4u_{iN_y - 1}^{n+1} - u_{iN_y - 2}^{n+1})/3, \\ v_{iN_y}^{n+1} = (4v_{iN_y - 1}^{n+1} - v_{iN_y - 2}^{n+1})/3, i = \overline{0, N_x}, \end{gathered} \tag{29}$$

Scheme for pulsation equations has the following form:

$$\frac{u'^{n+1}_{ij}-u'^{n}_{ij}}{\tau_{n+1}}+\frac{|u^n_{ij}|+u^n_{ij}}{2}u'^{n}_{\bar{x}}+\frac{u^n_{ij}-|u^n_{ij}|}{2}u'^{n}_{x}+\frac{|v^n_{ij}|+v^n_{ij}}{2}u'^{n}_{\bar{y}}+\frac{v^n_{ij}-|v^n_{ij}|}{2}u'^{n}_{y}+$$

$$+u'^{n}_{ij}u^n_{\tilde{x}}+v'^{n}_{ij}u^n_{\tilde{y}}+\frac{Dg}{2}[(u'^2)^n_{\tilde{x}}+(u'v')^n_{\tilde{y}}-(u'^2)^{n-1}_{\tilde{x}}-(u'v')^{n-1}_{\tilde{y}}]/\tau_n+$$

$$+\frac{p'^{n+1}_{i+1j}-p'^{n+1}_{ij}}{h_{xi+1}}=\frac{F'_x}{Fr}+\frac{2\mu^v_{uxv}}{h_{xi+1}+h_{xi}}(u'^{n}_{x}-u'^{n}_{\bar{x}})+\frac{2\mu^u_{vyu}}{h_{yj+1}+h_{yj}}(u'^{n}_{y}-u'^{n}_{\bar{y}})],$$

$$\frac{v'^{n+1}_{ij}-v'^{n}_{ij}}{\tau_{n+1}}+\frac{|u^n_{ij}|+u^n_{ij}}{2}v'^{n}_{\bar{x}}+\frac{u^n_{ij}-|u^n_{ij}|}{2}v'^{n}_{x}+\frac{|v^n_{ij}|+v^n_{ij}}{2}v'^{n}_{\bar{y}}+\frac{v^n_{ij}-|v^n_{ij}|}{2}v'^{n}_{y}+$$

$$+u'^{n}_{ij}v^n_{\tilde{x}}+v'^{n}_{ij}v^n_{\tilde{y}}+\frac{Dg}{2}[(u'v')^n_{\tilde{x}}+(v'^2)^n_{\tilde{y}}-(u'v')^{n-1}_{\tilde{x}}-(v'^2)^{n-1}_{\tilde{y}}]/\tau_n+$$

$$+\frac{p'^{n+1}_{ij+1}-p'^{n+1}_{ij}}{h_{yj+1}}=\frac{F'_y}{Fr}+\frac{2\mu^v_{uxv}}{h_{xi+1}+h_{xi}}(v'^{n}_{x}-v'^{n}_{\bar{x}})+\frac{2\mu^u_{vyu}}{h_{yj+1}+h_{yj}}(v'^{n}_{y}-v'^{n}_{\bar{y}})],$$

$$i=1,\ldots,N_x-1,\ j=1,\ldots,N_y-1;$$

$$\frac{u'^{n+1}_{ij}-u'^{n+1}_{i-1j}}{h_{xi}}+\frac{v'^{n+1}_{ij}-v'^{n+1}_{ij-1}}{h_{yj}}=0,\quad i=1,\ldots,N_x,\ j=1,\ldots,N_y;\ \ n=\overline{0,N_\tau}$$

§3. Semi-implicit scheme realization algorithm

Scheme realization algorithm (22)-(29) considerably simplifies, if notations of algebraic sums of convection and dissipative members are introduced into (22),(23):

$$Q^n_{uij}=\frac{2\mu^v_x}{h_{xi+1}+h_{xi}}(\frac{u^n_{i+1j}-u^n_{ij}}{h_{xi+1}}-\frac{u^n_{ij}-u^n_{i-1j}}{h_{xi}})+\frac{2\mu^u_y}{h_{yj+1}+h_{yj}}(\frac{u^n_{ij+1}-u^n_{ij}}{h_{yj+1}}-\frac{u^n_{ij}-u^n_{ij-1}}{h_{yj}})-$$

$$-\frac{|u_{ij}^n|+u_{ij}^n}{2}\frac{u_{ij}^n-u_{i-1j}^n}{h_{xi}}-\frac{u_{ij}^n-|u_{ij}^n|}{2}\frac{u_{i+1j}^n-u_{ij}^n}{h_{xi+1}}-$$

$$-\frac{|v_{ij}^n|+v_{ij}^n}{2}\frac{u_{ij}^n-u_{ij-1}^n}{h_{yj}}-\frac{v_{ij}^n-|v_{ij}^n|}{2}\frac{u_{ij+1}^n-u_{ij}^n}{h_{yj+1}}], \tag{30}$$

$$Q_{vij}^n=\frac{2\mu_x^v}{h_{xi+1}+h_{xi}}(\frac{v_{i+1j}^n-v_{ij}^n}{h_{xi+1}}-\frac{v_{ij}^n-v_{i-1j}^n}{h_{xi}})+\frac{2\mu_y^u}{h_{yj+1}+h_{yj}}(\frac{v_{ij+1}^n-v_{ij}^n}{h_{yj+1}}-\frac{v_{ij}^n-v_{ij-1}^n}{h_{yj}})-$$

$$-\frac{|u_{ij}^n|+u_{ij}^n}{2}\frac{v_{ij}^n-v_{i-1j}^n}{h_{xi}}-\frac{u_{ij}^n-|u_{ij}^n|}{2}\frac{v_{i+1j}^n-v_{ij}^n}{h_{xi+1}}-$$

$$-\frac{|v_{ij}^n|+v_{ij}^n}{2}\frac{v_{ij}^n-v_{ij-1}^n}{h_{yj}}-\frac{v_{ij}^n-|v_{ij}^n|}{2}\frac{v_{ij+1}^n-v_{ij}^n}{h_{yj+1}}] \tag{31}$$

Schemes (22),(23) are reduced to a standard form

$$\frac{u_{ij}^{n+1}-u_{ij}^n}{\tau_{n+1}}=\overline{Q}_{uij}^n-\frac{p_{i+1j}^{n+1}-p_{ij}^{n+1}}{h_{xi+1}}, \tag{32}$$

$$\frac{v_{ij}^{n+1}-v_{ij}^n}{\tau_{n+1}}=\overline{Q}_{vij}^n-\frac{p_{ij+1}^{n+1}-p_{ij}^{n+1}}{h_{yj+1}}, \tag{33}$$

where

$$\overline{Q}_{uij}^n=Q_{uij}^n+\frac{F_x}{Fr}-\{(u'^2)_{\tilde{x}}^n+(u'v')_{\tilde{y}}^n-$$

$$-\frac{Dg}{2}[(u'^2)_{\tilde{x}}^n+(u'v')_{\tilde{y}}^n-(u'^2)_{\tilde{x}}^{n-1}-(u'v')_{\tilde{y}}^{n-1}]/\tau_n\}, \tag{34}$$

$$\overline{Q}_{vij}^n=Q_{vij}^n+\frac{F_y}{Fr}-\{(u'v')_{\tilde{x}}^n+(v'^2)_{\tilde{y}}^n-$$

$$-\frac{Dg}{2}[(u'v')_{\tilde{x}}^n+(v'^2)_{\tilde{y}}^n-(u'v')_{\tilde{x}}^{n-1}-(v'^2)_{\tilde{y}}^{n-1}]/\tau_n\} \tag{35}$$

Both parts (32),(33) are multiplied by τ_{n+1} and are represented in the form

$$u_{ij}^{n+1} = u_{ij}^{n} + \tau_{n+1}\overline{Q}_{uij}^{n} - \tau_{n+1}\frac{p_{i+1j}^{n+1} - p_{ij}^{n+1}}{h_{xi+1}}, \tag{36}$$

$$v_{ij}^{n+1} = v_{ij}^{n} + \tau_{n+1}\overline{Q}_{vij}^{n} - \tau_{n+1}\frac{p_{ij+1}^{n+1} - p_{ij}^{n+1}}{h_{yj+1}} \tag{37}$$

In (36) and (37) let's introduce with the purpose of shortness,

$$F_{uij}^{n} = u_{ij}^{n} + \tau_{n+1}\overline{Q}_{uij}^{n}, \; F_{vij}^{n} = v_{ij}^{n} + \tau_{n+1}\overline{Q}_{vij}^{n} \quad , \tag{38}$$

and represent (36),(37) in even shorter form

$$u_{ij}^{n+1} = F_{uij}^{n} - \tau_{n+1}\frac{p_{i+1j}^{n+1} - p_{ij}^{n+1}}{h_{xi+1}}, \tag{39}$$

$$v_{ij}^{n+1} = F_{vij}^{n} - \tau_{n+1}\frac{p_{ij+1}^{n+1} - p_{ij}^{n+1}}{h_{yj+1}} \tag{40}$$

Absolutely similar (30) - (40) expressions are obtained for the scheme of pulsation equations as well (19), (20), (21):

$$\frac{u'^{n+1}_{ij} - u'^{n}_{ij}}{\tau_{n+1}} = \overline{Q}'^{n}_{uij} - \frac{p'^{n+1}_{i+1j} - p'^{n+1}_{ij}}{h_{xi+1}},$$

$$\frac{v'^{n+1}_{ij} - v'^{n}_{ij}}{\tau_{n+1}} = \overline{Q}'^{n}_{vij} - \frac{p'^{n+1}_{ij+1} - p'^{n+1}_{ij}}{h_{yj+1}},$$

where

$$\overline{Q}'^{n}_{uij} = Q'^{n}_{uij} + \frac{F_x'}{Fr} - u'^{n}_{ij}\, u^{n}_{\tilde{x}} - v'^{n}_{ij}\, u^{n}_{\tilde{y}} -$$

$$-\frac{Dg}{2}[(u'^2)^{n}_{\tilde{x}} + (u'v')^{n}_{\tilde{y}} - (u'^2)^{n-1}_{\tilde{x}} - (u'v')^{n-1}_{\tilde{y}}]/\tau_n,$$

$$\overline{Q}'^{n}_{vij} = Q'^{n}_{vij} + \frac{F_y'}{Fr} - u'^{n}_{ij}\, v^{n}_{\tilde{x}} - v'^{n}_{ij}\, v^{n}_{\tilde{y}} -$$

$$-\frac{Dg}{2}[(u'v')^{n}_{\tilde{x}} + (v'^2)^{n}_{\tilde{y}} - (u'v')^{n-1}_{\tilde{x}} - (v'^2)^{n-1}_{\tilde{y}}]/\tau_n$$

Having denoted

$$F'^{n}_{uij} = u'^{n}_{ij} + \tau_{n+1}\overline{Q}'^{n}_{uij}\,, \qquad F'^{n}_{vij} = v'^{n}_{ij} + \tau_{n+1}\overline{Q}'^{n}_{vij}\,,$$

we'll make a standard representation

$$u'^{n+1}_{ij} = F'^{n}_{uij} - \tau_{n+1}\frac{p'^{n+1}_{i+1j} - p'^{n+1}_{ij}}{h_{xi+1}},$$

$$v'^{n+1}_{ij} = F'^{n}_{vij} - \tau_{n+1}\frac{p'^{n+1}_{ij+1} - p'^{n+1}_{ij}}{h_{yj+1}}$$

There is a fundamental principle of realization of law of mass conservation via pressure function. By this principle pressure field p^{n+1}_{ij} is determined from requirement, that for those standing in (39) and (40) $u^{n+1}_{ij}, v^{n+1}_{ij}$ observed would be continuity equation (24). With this purpose (39),(40) are substituted into (24), and in the result obtained is the closed system of difference equations for pressure p^{n+1}_{ij}. Substitution is performed gradually.

§4. Technology of obtaining difference equations for pressure is explained for boundary conditions

$$u\big|_s = \varphi_u(x,y,t), v\big|_S = \varphi_v(x,y,t)$$

1^nd scheme

1^o. Substitution of (39) and (40) into (24) in nodes with numbers $2 \le i \le N_x - 1, 2 \le j \le N_y - 1$ gives a system of linear equations

$$\frac{1}{h_{xi}}\left[\left(F^{n}_{uij} - \tau_{n+1}\frac{p^{n+1}_{i+1j} - p^{n+1}_{ij}}{h_{xi+1}}\right) - \left(F^{n}_{ui-1j} - \tau_{n+1}\frac{p^{n+1}_{ij} - p^{n+1}_{i-1j}}{h_{xi}}\right)\right] +$$

$$+\frac{1}{h_{yj}}\left[\left(F^{n}_{vij} - \tau_{n+1}\frac{p^{n+1}_{ij+1} - p^{n+1}_{ij}}{h_{yj+1}}\right) - \left(F^{n}_{vij-1} - \tau_{n+1}\frac{p^{n+1}_{ij} - p^{n+1}_{ij-1}}{h_{yj}}\right)\right] = 0,$$

$$2 \le i \le N_x - 1, 2 \le j \le N_y - 1 \qquad (41)$$

2^o. Substitution of (39) and (40) into (24) in nodes with indices $i=1;\ 2\le j\le N_y-1$. In these nodes equation (24) has the following form:

$$\frac{u_{1j}^{n+1}-u_{0j}^{n+1}}{h_{x1}}+\frac{v_{1j}^{n+1}-v_{1j-1}^{n+1}}{h_{yj}}=0,\ 2\le j\le N_y-1, \qquad (42)$$

where boundary condition $u_{0j}^{n+1}=\varphi_{u0j}^{n+1}$ is set. After substitution the result is:

$$\frac{1}{h_{x1}}\left\{\left(F_{u1j}^{n}-\tau_{n+1}\frac{p_{2j}^{n+1}-p_{1j}^{n+1}}{h_{x2}}\right)-u_{0j}^{n+1}\right\}+\frac{1}{h_{yj}}\left\{\left(F_{v1j}^{n}-\right.\right. \qquad (43)$$

$$\left.\left.-\tau_{n+1}\frac{p_{1j+1}^{n+1}-p_{1j}^{n+1}}{h_{yj+1}}\right)-\left(F_{v1j-1}^{n}-\tau_{n+1}\frac{p_{1j}^{n+1}-p_{1j-1}^{n+1}}{h_{yj}}\right)\right\}=0, 2\le j\le N_y-1$$

***3^o*.** Similar substitution (39) and (40) into (24) is performed in nodes with indexes $j=1;\ 2\le i\le N_x-1,$ where (24) has the form

$$\frac{u_{i1}^{n+1}-u_{i-11}^{n+1}}{h_{xi}}+\frac{v_{i1}^{n+1}-v_{i0}^{n+1}}{h_{y1}}=0,\ 2\le i\le N_x-1, \qquad (44)$$

here, in accordance with boundary condition given is $v_{i0}^{n+1}=\varphi_{vi0}^{n+1}$. In the result

$$\frac{1}{h_{xi}}\left\{\left(F_{ui1}^{n}-\tau_{n+1}\frac{p_{i+11}^{n+1}-p_{i1}^{n+1}}{h_{xi+1}}\right)-\left(F_{ui-11}^{n}-\tau_{n+1}\frac{p_{i1}^{n+1}-p_{i-11}^{n+1}}{h_{xi}}\right)\right\}+$$

$$+\frac{1}{h_{y1}}\left\{\left(F_{vi1}^{n}-\tau_{n+1}\frac{p_{i2}^{n+1}-p_{i1}^{n+1}}{h_{y2}}\right)-v_{i0}^{n+1}\right\}=0, 2\le i\le N_x-1 \qquad (45)$$

***4^o*.** Substitution of (39) and (40) into (24) in corner node with indexes $i=1;\ \ j=1;$ where (24) has the form

$$\frac{u_{11}^{n+1}-u_{01}^{n+1}}{h_{x1}}+\frac{v_{11}^{n+1}-v_{10}^{n+1}}{h_{y1}}=0,$$

results in expression:

$$\frac{1}{h_{x1}}\left\{F_{u11}^{n}-\tau_{n+1}\frac{p_{21}^{n+1}-p_{11}^{n+1}}{h_{x2}})-u_{01}^{n+1}\right\}$$

$$+\frac{1}{h_{y1}}\left\{F_{v11}^{n}-\tau_{n+1}\frac{p_{12}^{n+1}-p_{11}^{n+1}}{h_{y2}})-v_{10}^{n+1}\right\}0 \qquad (46)$$

In (41), (43),(45),(46) substitution in internal nodes Ω_h was carried out, the resulted system of linear equations for p_{ij}^{n+1} is open-ended. For closing this system, engaged is continuity equation (24) in boundary nodes γ_h with indices $i=N_x$, $1\le j\le N_y-1$: $j=N_y$, $2\le i\le N_x-1$.

5^o. In boundary nodes $i=N_x$, $1\le j\le N_y-1$ continuity equation (24) has the form

$$\frac{u_{N_xj}^{n+1}-u_{N_x-1j}^{n+1}}{h_{xN_x}}+\frac{v_{N_xj}^{n+1}-v_{N_xj-1}^{n+1}}{h_{yj}}=0,\ 1\le j\le N_y-1, \qquad (47)$$

where by boundary conditions the following considered known

$$u_{N_xj}^{n+1}=\varphi_{uN_xj}^{n+1}, v_{N_xj}^{n+1}=\varphi_{vN_xj}^{n+1}, v_{N_xj-1}^{n+1}=\varphi_{vN_xj-1}^{n+1}, j=\overline{0,N_y-1},$$

accordingly, only (39) is substituted instead of $u_{N_x-1j}^{n+1}$. As the result obtained is

$$\frac{1}{h_{xN_x}}\{u_{N_xj}^{n+1}-(F_{uN_x-1j}^{n}-\tau_{n+1}\frac{p_{N_xj}^{n+1}-p_{N_x-1j}^{n+1}}{h_{xN_x}})\}+\frac{v_{N_xj}^{n+1}-v_{N_xj-1}^{n+1}}{h_{yj}}=0, \qquad (48)$$

$$1\le j\le N_y-1$$

6^o. In boundary nodes $j=N_y$, $1\le i\le N_x-1$ continuity equation (24) has the form

$$\frac{u_{iN_y}^{n+1}-u_{i-1N_y}^{n+1}}{h_{xi}}+\frac{v_{iN_y}^{n+1}-v_{iN_y-1}^{n+1}}{h_{yN_y}}=0,\quad 1\le i\le N_x-1,$$

where by boundary conditions the following considered known

$$u_{iN_y}^{n+1} = \varphi_{uiN_y}^{n+1}, v_{iN_y}^{n+1} = \varphi_{viN_y}^{n+1}, u_{i-1N_y}^{n+1} = \varphi_{ui-1N_y}^{n+1}, i = 0,\ldots,N_x,$$

therefore (40) is substituted instead of $v_{iN_y-1}^{n+1}$. As the result obtained is

$$\frac{u_{iN_y}^{n+1} - u_{i-1N_y}^{n+1}}{h_{xi}} + \frac{1}{h_{yN_y}}\{v_{iN_y}^{n+1} - (F_{viN_y-1}^{n} - \tau_{n+1}\frac{p_{iN_y}^{n+1} - p_{iN_y-1}^{n+1}}{h_{yj}})\} = 0, 1 \le i \le N_x - 1 \quad (49)$$

Difference equations (48) and (49) are natural boundary conditions for pressure. System (41),(43),(45),(46),(48),(49) is the closed system, which is solved by iterative method, for applying of which it is convenient to represent equations (41),(43),(45),(46) in the form of one formula using signature function $sign(x)$:

$sign(x)=1$ for $x > 0$, $sign(x)=0$ for $x = 0$, $sign(x) = -1$ for $x < 0$,

$$\frac{1}{h_{xi}}\left[(F_{uij}^{n} - \tau_{n+1}\frac{p_{i+1j}^{n+1} - p_{ij}^{n+1}}{h_{xi+1}}) - \frac{1+sign(i-1.5)}{2}(F_{ui-1j}^{n} -\right.$$
$$\left.-\tau_{n+1}\frac{p_{ij}^{n+1} - p_{i-1j}^{n+1}}{h_{xi}}) - \frac{1-sign(i-1.5)}{2}u_{0j}^{n+1}\right] + \frac{1}{h_{yj}}\left[(F_{vij}^{n} -\right.$$
$$-\tau_{n+1}\frac{p_{ij+1}^{n+1} - p_{ij}^{n+1}}{h_{yj+1}}) - \frac{1+sign(j-1.5)}{2}(F_{vij-1}^{n} - \tau_{n+1}\frac{p_{ij}^{n+1} - p_{ij-1}^{n+1}}{h_{yj}}) -$$
$$\left.-\frac{1-sign(j-1.5)}{2}v_{i0}^{n+1}\right] = 0,\ 1 \le i \le N_x - 1, 1 \le j \le N_y - 1 \quad (50)$$

System (50) is solved in relation to p_{ij}^{n+1} with boundary conditions (48), (49).

2nd scheme with forwardwise difference derivatives in continuity equation ($\overline{\alpha} = 1, \overline{\beta} = 0$ /1/):

$$\frac{u_{i+1j}^{n+1} - u_{ij}^{n+1}}{h_{xi+1}} + \frac{v_{ij+1}^{n+1} - v_{ij}^{n+1}}{h_{yj+1}} = 0,\ 0 \le i \le N_x - 1, 0 \le j \le N_y - 1 \quad (51)$$

In this case pressure gradients in equations (16) and (17), /2/, should be approximated by backward differences,

$$u_{ij}^{n+1} = F_{uij}^{n} - \tau_{n+1}\frac{p_{ij}^{n+1} - p_{i-1j}^{n+1}}{h_{xi}}, \tag{52}$$

$$v_{ij}^{n+1} = F_{vij}^{n} - \tau_{n+1}\frac{p_{ij}^{n+1} - p_{ij-1}^{n+1}}{h_{yj}} \tag{53}$$

In the result of similar ***1^o,2^o,3^o,4^o,5^o,6^o*** substitutions (52) and (53) into (51) obtained is system of equations for pressure:

$$\Big\{(F_{ui+1j}^{n} - \tau_{n+1}\frac{p_{i+1j}^{n+1} - p_{ij}^{n+1}}{h_{xi+1}})\frac{1-sign(i-N_x+1.5)}{2} +$$

$$+\frac{1+sign(i-N_x+1.5)}{2}u_{N_x j}^{n+1} - (F_{uij}^{n} - \tau_{n+1}\frac{p_{ij}^{n+1} - p_{i-1j}^{n+1}}{h_{xi}})\Big\}\frac{1}{h_{xi+1}} +$$

$$+\Big\{\frac{1+sign(j-N_y+1.5)}{2}v_{iN_y}^{n+1} +$$

$$+(F_{vij+1}^{n} - \tau_{n+1}\frac{p_{ij+1}^{n+1} - p_{ij}^{n+1}}{h_{yj+1}})\frac{1-sign(j-N_y+1.5)}{2} -$$

$$-(F_{vij}^{n} - \tau_{n+1}\frac{p_{ij}^{n+1} - p_{ij-1}^{n+1}}{h_{yj}})\Big\}\frac{1}{h_{yj+1}} = 0,$$

$$i = 1,\ldots,N_x - 1,\ j = 1,\ldots,N_y - 1;$$

$$\frac{1}{h_{x1}}\{(F_{u1j}^{n} - \tau_{n+1}\frac{p_{1j}^{n+1} - p_{0j}^{n+1}}{h_{x1}}) - \varphi_{u0j}^{n+1}\} + (\frac{\partial\varphi_v}{\partial y})_{0j}^{n+1} = 0,$$

$$i = 0,\ j = 1,\ldots,N_y - 1;$$

$$(\frac{\partial\varphi_u}{\partial x})_{i0}^{n+1} + \frac{1}{h_{y1}}\{(F_{vi1}^{n} - \tau_{n+1}\frac{p_{i1}^{n+1} - p_{i0}^{n+1}}{h_{y1}}) - \varphi_{vi0}^{n+1}\} = 0,$$

$$j = 0,\ i = 1,\ldots,N_x - 1$$

3^rd^ scheme with central difference derivatives in continuity equation in nodes Ω_h

$$\frac{u_{i+1j}^{n+1}-u_{i-1j}^{n+1}}{h_{xi+1}+h_{xi}}+\frac{v_{ij+1}^{n+1}-v_{ij-1}^{n+1}}{h_{yj+1}+h_{yj}}=0,\ 1\le i\le N_x-1, 1\le j\le N_y-1 \qquad (54)$$

The following schemes are used in boundary nodes

$$\frac{u_{N_xj}^{n+1}-u_{N_x-1j}^{n+1}}{h_{xN_x}}+\frac{v_{N_xj}^{n+1}-v_{N_xj-1}^{n+1}}{h_{yj}}=0,\ \frac{u_{1j}^{n+1}-u_{0j}^{n+1}}{h_{x1}}+\frac{v_{0j+1}^{n+1}-v_{0j}^{n+1}}{h_{yj+1}}=0, \qquad (55)$$

$$\frac{u_{iN_y}^{n+1}-u_{i-1N_y}^{n+1}}{h_{xi}}+\frac{v_{iN_y}^{n+1}-v_{iN_y-1}^{n+1}}{h_{yN_y}}=0,\ \frac{u_{i+10}^{n+1}-u_{i0}^{n+1}}{h_{xi+1}}+\frac{v_{i1}^{n+1}-v_{i0}^{n+1}}{h_{y1}}=0,$$

by principle of interconsistent approximation /1/ central differences are used for pressure

$$u_{ij}^{n+1}=F_{uij}^{n}-\tau_{n+1}\frac{p_{i+1j}^{n+1}-p_{i-1j}^{n+1}}{h_{xi+1}+h_{xi}},, \qquad (56)$$

$$v_{ij}^{n+1}=F_{vij}^{n}-\tau_{n+1}\frac{p_{ij+1}^{n+1}-p_{ij-1}^{n+1}}{h_{yj+1}+h_{yj}} \qquad (57)$$

Equations for p_{ij}^{n+1} are derived by similar to ***[1]*** method:

$$\{(F_{ui+1j}^{n}-\tau_{n+1}\frac{p_{i+2j}^{n+1}-p_{ij}^{n+1}}{h_{xi+2}+h_{xi+1}})\frac{1-sign(i-N_x+1.5)}{2}+$$

$$+\frac{1+sign(i-N_x+1.5)}{2}u_{N_xj}^{n+1}-\frac{1+sign(i-1.5)}{2}(F_{ui-1j}^{n}-\tau_{n+1}\frac{p_{ij}^{n+1}-p_{i-2j}^{n+1}}{h_{xi}+h_{xi-1}})-$$

$$-\frac{1-sign(i-1.5)}{2}u_{0j}^{n+1}\}\frac{1}{h_{xi}+h_{xi+1}}+\{\frac{1+sign(j-N_y+1.5)}{2}v_{iN_y}^{n+1}+$$

$$+(F_{vij+1}^{n}-\tau_{n+1}\frac{p_{ij+2}^{n+1}-p_{ij}^{n+1}}{h_{yj+2}+h_{yj+1}})\frac{1-sign(j-N_y+1.5)}{2}-$$

$$-(F_{vij-1}^{n}-\tau_{n+1}\frac{p_{ij}^{n+1}-p_{ij-2}^{n+1}}{h_{yj}+h_{yj-1}})\frac{1+sign(j-1.5)}{2}-$$

$$-\frac{1-sign(j-1.5)}{2}v_{i0}^{n+1}\}\frac{1}{h_{yj}+h_{yj+1}}=0,\ i=1,\ldots,N_x-1,\ j=1,\ldots,N_y-1;$$

boundary conditions for pressure are derived from (55):

$$\frac{1}{h_{xN_x}}\{\varphi_{uN_xj}^{n+1}-(F_{uN_x-1j}^{n}-\tau_{n+1}\frac{p_{N_xj}^{n+1}-p_{N_x-2j}^{n+1}}{h_{xN_x}+h_{xN_x-1}})\}+(\frac{\partial\varphi_v}{\partial y})_{N_xj}^{n+1}=0,$$

$$i=N_x,\ j=1,\ldots,N_y-1;$$

$$\frac{1}{h_{x1}}\{(F_{u1j}^{n}-\tau_{n+1}\frac{p_{2j}^{n+1}-p_{0j}^{n+1}}{h_{x2}+h_{x1}})-\varphi_{u0j}^{n+1}\}+(\frac{\partial\varphi_v}{\partial y})_{0j}^{n+1}=0,$$

$$i=0,\ j=1,\ldots,N_y-1;$$

$$(\frac{\partial\varphi_u}{\partial x})_{iN_y}^{n+1}+\frac{1}{h_{yN_y}}\{\varphi_{viN_y}^{n+1}-(F_{viN_y-1}^{n}-\tau_{n+1}\frac{p_{iN_y}^{n+1}-p_{iN_y-2}^{n+1}}{h_{yN_y}+h_{yN_y-1}})\}=0,$$

$$j=N_y,\ i=1,\ldots,N_x-1,$$

$$(\frac{\partial\varphi_u}{\partial x})_{i0}^{n+1}+\frac{1}{h_{y1}}\{(F_{vi1}^{n}-\tau_{n+1}\frac{p_{i2}^{n+1}-p_{i0}^{n+1}}{h_{y2}+h_{y1}})-\varphi_{vi0}^{n+1}\}=0,$$

$$j=0,\ i=1,\ldots,N_x-1$$

§5. Iteration algorithm. Global iterations method

Solution of system of linear algebraic equations (48),(49),(50) in relation to p_{ij}^{n+1} may be obtained by modified Jacobi method (simple iteration). For simplification of representing the algorithm upper

index "$n+1$" in p_{ij}^{n+1} is omitted: $p_{ij}=p_{ij}^{n+1}$. Number of κ-iteration is denoted as p_{ij}^{k}. For convergent iterative process

$$\lim_{k\to\infty} p_{ij}^{k} = p_{ij}^{n+1}, \forall (i, j) \tag{58}$$

Zero iteration p_{ij}^{0} when $k=0$ on each time layer t_{n+1} is equal to value of pressure on preceding time layer t_n :

$$p_{ij}^{0}=p_{ij}^{n}, \quad \forall (i, j) \tag{59}$$

$$\begin{aligned}
&\frac{p_{ij}^{k+1}-p_{ij}^{k}}{\theta}+\frac{1}{h_{xi}}\Big((F_{uij}^{n}-\tau_{n+1}\frac{p_{i+1j}^{k}-p_{ij}^{k+1}}{h_{xi+1}})-\\
&-\frac{1+sign(i-1.5)}{2}(F_{ui-1j}^{n}-\tau_{n+1}\frac{p_{ij}^{k+1}-p_{i-1j}^{k}}{h_{xi}})-\\
&-\frac{1-sign(i-1.5)}{2}u_{0j}^{n+1}\Big)+\frac{1}{h_{yj}}\Big((F_{vij}^{n}-\tau_{n+1}\frac{p_{ij+1}^{k}-p_{ij}^{k+1}}{h_{yj+1}})-\\
&-\frac{1+sign(j-1.5)}{2}(F_{vij-1}^{n}-\tau_{n+1}\frac{p_{ij}^{k+1}-p_{ij-1}^{k}}{h_{yj}})-\\
&-\frac{1-sign(j-1.5)}{2}v_{i0}^{n+1}\Big)=0, \quad 1\le i\le N_x-1, 1\le j\le N_y-1
\end{aligned} \tag{60}$$

Iteration algorithm for boundary condition (49):

$$\begin{aligned}
&\frac{p_{iN_y}^{k+1}-p_{iN_y}^{k}}{\theta}+\frac{u_{iN_y}^{n+1}-u_{i-1N_y}^{n+1}}{h_{xi}}+\frac{1}{h_{yN_y}}\{v_{iN_y}^{n+1}-(F_{viN_y-1}^{n}-\\
&-\tau_{n+1}\frac{p_{iN_y}^{k+1}-p_{iN_y-1}^{k}}{h_{yN_y}})\}=0, \quad 1\le i\le N_x-1 ,
\end{aligned} \tag{61}$$

where coefficient with $p_{iN_y}^{k+1}$ is equal to $A_{iN_y}^{n} = \dfrac{\tau_{n+1}}{h_{yN_y} h_{yN_y}}$. Iteration algorithm for boundary condition (48):

$$\frac{p_{N_x j}^{k+1} - p_{N_x j}^{k}}{\theta} + \frac{1}{h_{xN_x}}\{u_{N_x j}^{n+1} - (F_{uN_x-1j}^{n} - \tau_{n+1}\frac{p_{N_x j}^{k+1} - p_{N_x-1j}^{k}}{h_{xN_x}})\} +$$

$$+\frac{v_{N_x j}^{n+1} - v_{N_x j-1}^{n+1}}{h_{yj}} = 0,\ 1 \le j \le N_y - 1\ , \qquad (62)$$

where coefficient with $p_{N_x j}^{k+1}$ is equal to $A_{N_x j}^{n} = \dfrac{\tau_{n+1}}{h_{xN_x} h_{xN_x}}$.

Here, $0 < \theta < 1 -$ iteration parameter. In (*60*) collected are coefficients with p_{ij}^{k+1} in the form

$$A_{ij}^{n} = \tau_{n+1}\{\left[\frac{1}{h_{xi}}\left(\frac{1}{h_{xi+1}} + \frac{1+sign(i-1.5)}{2h_{xi}}\right)\right] +$$

$$+\left[\frac{1}{h_{yj}}\left(\frac{1}{h_{yj+1}} + \frac{1+sign(j-1.5)}{2h_{yj}}\right)\right]\} \qquad (63)$$

for explicit definition

$$p_{ij}^{k+1} = (\frac{1}{\theta} + A_{ij}^{n})^{-1}\left\langle \frac{p_{ij}^{k}}{\theta} - \frac{1}{h_{xi}}\{(F_{uij}^{n} - \tau_{n+1}\frac{p_{i+1j}^{k}}{h_{xi+1}}) - \right.$$

$$-\frac{1+sign(i-1.5)}{2}(F_{ui-1j}^{n} + \tau_{n+1}\frac{p_{i-1j}^{k}}{h_{xi}}) - \frac{1-sign(i-1.5)}{2}u_{0j}^{n+1}\}$$

$$-\frac{1}{h_{yj}}\{(F_{vij}^{n} - \tau_{n+1}\frac{p_{ij+1}^{k}}{h_{yj+1}}) - \frac{1+sign(j-1.5)}{2} * \qquad (64)$$

$$* \ (F_{vij-1}^{n} + \tau_{n+1} \frac{p_{ij-1}^{k}}{h_{yj}}) - \frac{1 - sign(j-1.5)}{2} v_{i0}^{n+1}\} \Big\rangle,$$

Similarly, from (61) and (62) computed are

$$p_{iN_y}^{k+1} = \frac{1}{\frac{1}{\theta} + A_{iN_y}^{n}} \left\{ \frac{p_{iN_y}^{k}}{\theta} - \frac{u_{iN_y}^{n+1} - u_{i-1N_y}^{n+1}}{h_{xi}} - \frac{1}{h_{yN_y}} \left\{ v_{iN_y}^{n+1} - (F_{viN_y-1}^{n} - \right.\right.$$

$$\left. + \tau_{n+1} \frac{p_{iN_y-1}^{k}}{h_{yN_y}}) \right\} \ 1 \le i \le N_x - 1 \ , \quad (65)$$

$$p_{N_xj}^{k+1} = \frac{1}{\frac{1}{\theta} + A_{N_xj}^{n}} \left\{ \frac{p_{N_xj}^{k}}{\theta} - \frac{1}{h_{xN_x}} \ (u_{N_xj}^{n+1} - \right. \quad (66)$$

$$\left. - (F_{uN_x-1j}^{n} + \tau_{n+1} \frac{p_{N_x-1j}^{k}}{h_{xN_x}})\} - \frac{v_{N_xj}^{n+1} - v_{N_x-1j}^{n+1}}{h_{yj}} \right\}, \quad 1 \le j \le N_y - 1$$

Iterations (64),(65),(66) are continued until realization of continuity equation (24) with accuracy $\varepsilon \approx 0$. Obviously, equations (24) and (50) are equivalent to each other because of representations (39) and (40), i.e. equation (50) is also a continuity equation, but represented in other manner. This circumstance allows to considerably simplify procedure of iteration algorithm for computation of p_{ij}^{n+1}. With this purpose (60) with the use of coefficient A_{ij}^{n} is re-represented in the form

$$(1 + \theta A_{ij}^{n}) \frac{p_{ij}^{k+1} - p_{ij}^{k}}{\theta} + \Big\langle \ \frac{1}{h_{xi}} \ (F_{uij}^{n} - \tau_{n+1} \frac{p_{i+1j}^{k} - p_{ij}^{k}}{h_{xi+1}}) -$$

$$
\begin{gathered}
-\frac{1+sign(-1.5)}{2}(F^{n}_{ui-1j}-\\
-\tau_{n+1}\frac{p^{k}_{ij}-p^{k}_{i-1j}}{h_{xi}})-\frac{1-sign(i-1.5)}{2}u^{n+1}_{0j}\}+\\
+\frac{1}{h_{yj}}\{(F^{n}_{vij}-\tau_{n+1}\frac{p^{k}_{ij+1}-p^{k}_{ij}}{h_{yj+1}})-\\
-\frac{1+sign(j-1.5)}{2}*(F^{n}_{vij-1}-\tau_{n+1}\frac{p^{k}_{ij}-p^{k}_{ij-1}}{h_{yj}})-\\
-\frac{1-sign(j-1.5)}{2}v^{n+1}_{i0}\}\Big\rangle=0
\end{gathered}
\tag{67}
$$

Global iterations method for (24),(39),(40) in the first scheme

Obviously expression in the left part (67), in brackets $\Big\langle \quad \dots \Big\rangle$, matches with (50), and (50) matches with continuity equation (24). Basing on this, it is convenient to introduce grid functions $u^{n+1^k}_{ij}, v^{n+1^k}_{ij}$ by analogy with (39),(40):

$$
u^{n+1^k}_{ij}=F^{n}_{uij}-\tau_{n+1}\frac{p^{k}_{i+1j}-p^{k}_{ij}}{h_{xi+1}}, \tag{68}
$$

$$
v^{n+1^k}_{ij}=F^{n}_{vij}-\tau_{n+1}\frac{p^{k}_{ij+1}-p^{k}_{ij}}{h_{yj+1}}, \tag{69}
$$

at this $u^{n+1^k}_{ij}, v^{n+1^k}_{ij}$ are *k*-iterations,

accordingly, $u^{n+1}_{ij}, v^{n+1}_{ij}$, that is, in the limit

$$\lim_{k\to\infty} u_{ij}^{n+1^k} = u_{ij}^{n+1}\ ,\ \lim_{k\to\infty} v_{ij}^{n+1^k} = v_{ij}^{n+1}\ ,$$

since in iterative convergence (64),(65),(66)

$$\lim_{k\to\infty} p_{ij}^{k} = p_{ij}^{n+1}, \forall (i, j)$$

Due to above-indicated match of expression in the left part (*67*), standing in brackets $\langle\ \ldots\ \rangle$ with (50), and (50) with continuity equation (24), contained $\langle\ \ldots\ \rangle$ in (*67*) is replaced for iterative equation

$$(1+\theta A_{ij}^{n})\frac{p_{ij}^{k+1}-p_{ij}^{k}}{\theta}+\frac{u_{ij}^{n+1^k}-u_{i-1j}^{n+1^k}}{h_{xi}}+$$

$$+\frac{v_{ij}^{n+1^k}-v_{ij-1}^{n+1^k}}{h_{yj}}=0,1\le i\le N_x,1\le j\le N_y \qquad (70)$$

System (70) is equivalent to (67) and boundary conditions (65) and (*66*), is solved in relation to p_{ij}^{k+1} , where by boundary conditions (26),(27),(28),(29):

$$x=0: u_{0j}^{n+1^k}=e^{-\frac{b^o}{t_{n+1}}}, v_{0j}^{n+1^k}=0,\ j=\overline{0, N_y-1},$$

$$x=1+2l/L, u_{N_xj}^{n+1^k}=2u_{N_x-1j}^{n+1^k}-u_{N_x-2j}^{n+1^k},$$

$$v_{N_xj}^{n+1^k}=2v_{N_x-1j}^{n+1^k}-v_{N_x-2j}^{n+1^k}, 0\le j\le N_y;$$

$$on \quad plate \quad y=0, u_{i0}^{n+1^k}=0, v_{i0}^{n+1^k}=0, mk\le i\le mkk, \qquad (71)$$

before the beginning and after the end of plate

$$y=0, u_{i0}^{n+1^k}=(4u_{i1}^{n+1^k}-u_{i2}^{n+1^k})/3, v_{i0}^{n+1^k}=0,$$

$$1\le i\le mk-1, mkk+1\le i\le N_x,$$

$$y = H/L: \quad u_{iN_y}^{n+1^k} = (4u_{iN_y-1}^{n+1^k} - u_{iN_y-2}^{n+1^k})/3,$$

$$v_{iN_y}^{n+1^k} = (4v_{iN_y-1}^{n+1^k} - v_{iN_y-2}^{n+1^k})/3, i = \overline{0, N_x}$$

Iteration process (68) - (71-73) is absolutely identical (64), (65),(66) and is realized all together until such value of $k = \bar{k}$, with which continuity equation (24) in grid nodes is performed with the prescribed accuracy $\varepsilon \approx 0$:

$$\left| \frac{u_{ij}^{n+1^k} - u_{i-1j}^{n+1^k}}{h_{xi}} + \frac{v_{ij}^{n+1^k} - v_{ij-1}^{n+1^k}}{h_{yj}} \right| \leq \varepsilon, \forall (i, j) \qquad (72)$$

Iteration process is realized until criterion (72) is fulfilled. In all cases in satisfying inequalities (72) for solution on time layer t_{n+1} taken is

$$p_{ij}^{\bar{k}+1} = p_{ij}^{n+1}, u_{ij}^{n+1^{\bar{k}}} = u_{ij}^{n+1}, v_{ij}^{n+1^{\bar{k}}} = v_{ij}^{n+1}, \forall (i, j)$$

Global iterations method for (54)-(55)-(56)-(57)

when p_{ij}^{n+1}, coefficient in the scheme with central differences:

$$A_{ij} = \tau_{n+1}\{\frac{1}{h_{xi+1} + h_{xi}}[\frac{1}{h_{xi+2} + h_{xi+1}} \frac{1 - sign(i - N_x + 1.5)}{2} +$$

$$+ \frac{1}{h_{xi} + h_{xi-1}} \frac{1 + sign(i - 1.5)}{2}] +$$

$$+ \frac{1}{h_{yj+1} + h_{yj}}[\frac{1}{h_{yj+2} + h_{yj+1}} \frac{1 - sign(j - N_y + 1.5)}{2} +$$

$$+ \frac{1}{h_{yj} + h_{yj-1}} \frac{1 + sign(j - 1.5)}{2}]\}, i = 1, \ldots, N_x - 1, j = 1, \ldots, N_y - 1;$$

$$A_{N_x j} = \tau_{n+1} \frac{1}{h_{xN_x}(h_{xN_x} + h_{xN_x-1})},$$

$$A_{0j} = \tau_{n+1} \frac{1}{h_{x1}(h_{x2} + h_{x1})}, j = 1,\ldots,N_y - 1;$$

$$A_{iN_y} = \tau_{n+1} \frac{1}{h_{yN_y}(h_{yN_y} + h_{yN_y-1})},$$

$$A_{i0} = \tau_{n+1} \frac{1}{h_{y1}(h_{y2} + h_{y1})}, i = 1,\ldots,N_x - 1$$

Global iteration process is built as follows:

$$u_{ij}^{n+1^k} = F_{uij}^n - \tau_{n+1} \frac{p_{i+1j}^k - p_{i-1j}^k}{h_{xi+1} + h_{xi}},$$

$$v_{ij}^{n+1^k} = F_{vij}^n - \tau_{n+1} \frac{p_{ij+1}^k - p_{ij-1}^k}{h_{yj+1} + h_{yj}},$$

$$i = 1,\ldots,N_x - 1,\ j = 1,\ldots,N_y - 1;$$

$$(1 + A_{ij}\theta)\frac{p_{ij}^{k+1} - p_{ij}^k}{\theta} + \frac{u_{i+1j}^{n+1^k} - u_{i-1j}^{n+1^k}}{h_{xi+1} + h_{xi}} + \frac{v_{ij+1}^{n+1^k} - v_{ij-1}^{n+1^k}}{h_{yj+1} + h_{yj}} = 0,$$

$$1 \le i \le N_x - 1, 1 \le j \le N_y - 1$$

$$(1 + A_{N_x j}\theta)\frac{p_{N_x j}^{k+1} - p_{N_x j}^k}{\theta} + \frac{\varphi_{uN_x j}^{n+1} - u_{N_x-1j}^{n+1^k}}{h_{xN_x}} + (\frac{\partial \varphi_v}{\partial y})_{N_x j}^{n+1} = 0,$$

$$(1 + A_{0j}\theta)\frac{p_{0j}^{k+1} - p_{0j}^k}{\theta} + \frac{u_{1j}^{n+1^k} - \varphi_{u0j}^{n+1}}{h_{x1}} + (\frac{\partial \varphi_v}{\partial y})_{0j}^{n+1} = 0,$$

$$1 \le j \le N_y - 1,$$

$$(1+A_{iN_y}\theta)\frac{p_{iN_y}^{k+1}-p_{iN_y}^{k}}{\theta}+(\frac{\partial\varphi_u}{\partial x})_{iN_y}^{n+1}+\frac{\varphi_{viN_y}^{n+1}-v_{iN_y-1}^{n+1,k}}{h_{yN_y}}=0,$$

$$(1+A_{i0}\theta)\frac{p_{i0}^{k+1}-p_{i0}^{k}}{\theta}+(\frac{\partial\varphi_u}{\partial x})_{i0}^{n+1}+\frac{v_{i1}^{n+1,k}-\varphi_{vi0}^{n+1}}{h_{y1}}=0,$$

$$1\le i\le N_x-1$$

Iteration process is topped when condition (72) is fulfilled $\left|\frac{u_{ij}^{n+1^k}-u_{i-1j}^{n+1^k}}{h_{xi}}+\frac{v_{ij}^{n+1^k}-v_{ij-1}^{n+1^k}}{h_{yj}}\right|\le\varepsilon$

After that, last approximations $p_{ij}^{n+1,k^*}, u_{ij}^{n+1,k^*}, v_{ij}^{n+1,k^*}$ are taken as a solution $p_{ij}^{k^*}\approx p_{ij}^{n+1}, u_{ij}^{n+1^{k^*}}\approx u_{ij}^{n+1}, v_{ij}^{n+1^{k^*}}\approx v_{ij}^{n+1}$.

Iteration parameter $\theta>0$ may take any value from interval $0<\theta\le 1/2$.

Global iterations method for (51),(52),(53) in the 2nd scheme

$$u_{ij}^{n+1^k}=F_{uij}^{n}-\tau_{n+1}\frac{p_{ij}^{k}-p_{i-1j}^{k}}{h_{xi}},$$

$$v_{ij}^{n+1^k}=F_{vij}^{n}-\tau_{n+1}\frac{p_{ij}^{k}-p_{ij-1}^{k}}{h_{yj}},$$

$$(1+A_{ij}\theta)\frac{p_{ij}^{k+1}-p_{ij}^{k}}{\theta}+\frac{u_{i+1j}^{n+1^k}-u_{ij}^{n+1^k}}{h_{xi+1}}+\frac{v_{ij+1}^{n+1^k}-v_{ij}^{n+1^k}}{h_{yj+1}}=0,$$

$$0 \le i \le N_x - 1, 0 \le j \le N_y - 1$$

***Global iterations for (54)-(55)-(56)-(57) by Krasnoselsky - Krein method of minimal residuals* (m.m.r.)**

This is a universal method. Solution of a system of linear algebraic equations

$$\sum_{j=1}^{N} a_{ij} x_j = b_i, i = 1,\ldots,N ,$$

is found by iteration algorithm

$$x_i^{k+1} = x_i^k \ + \theta_k R_i^k, i = 1,\ldots,N \ , \ \ k = 0,1,\ldots,k^* ,$$

where R_i^k - residual vector of k-iteration

$$R_i^k = \sum_{j=1}^{N} a_{ij} x_j^k \ - b_i, i = 1,\ldots,N ,$$

θ_k=const – iteration parameter, is selected from condition of minimization of residual *k+1*—iteration $\sum_{i=1}^{n} (R_i^{k+1})^2$ and is equal

$$\theta_k = -\sum_{i=1}^{N} (\sum_{j=1}^{N} a_{ij} R_j^k) R_i^k \Big/ (\sum_{j=1}^{N} a_{ij} R_j^k) * (\sum_{j=1}^{N} a_{ij} R_j^k)$$

Iterations are stopped when criterion $\max\limits_{1 \le i \le N} \left| R_i^{k^*} \right| \le \varepsilon$ is fulfilled, where $0 < \varepsilon$ - sufficiently small number.

According to (m.m.r.) by (54)-(55) calculated are residuals of k-iterations

$$u_{ij}^{n+1^k} = F_{uij}^n - \tau_{n+1} \frac{p_{i+1j}^k - p_{i-1j}^k}{h_{xi+1} + h_{xi}},$$

$$v_{ij}^{n+1^k} = F_{vij}^n - \tau_{n+1} \frac{p_{ij+1}^k - p_{ij-1}^k}{h_{yj+1} + h_{yj}},$$

$$i = 1,\ldots,N_x - 1, \ j = 1,\ldots,N_y - 1;$$

$$R_{ij}^{k}=\frac{u_{i+1j}^{n+1^{k}}-u_{i-1j}^{n+1^{k}}}{h_{xi+1}+h_{xi}}+\frac{v_{ij+1}^{n+1^{k}}-v_{ij-1}^{n+1^{k}}}{h_{yj+1}+h_{yj}},$$

$$1\le i\le N_{x}-1,1\le j\le N_{y}-1,$$

$$R_{N_{x}j}^{k}=\frac{\varphi_{uN_{x}j}^{n+1}-u_{N_{x}-1j}^{n+1^{k}}}{h_{xN_{x}}}+(\frac{\partial\varphi_{v}}{\partial y})_{N_{x}j}^{n+1},1\le j\le N_{y}-1,$$

$$R_{0j}^{k}=\frac{u_{1j}^{n+1^{k}}-\varphi_{u0j}^{n+1}}{h_{x1}}+(\frac{\partial\varphi_{v}}{\partial y})_{0j}^{n+1},1\le j\le N_{y}-1,$$

$$R_{iN_{y}}^{k}=(\frac{\partial\varphi_{u}}{\partial x})_{iN_{y}}^{n+1}+\frac{\varphi_{viN_{y}}^{n+1}-v_{iN_{y}-1}^{n+1,k}}{h_{yN_{y}}},1\le i\le N_{x}-1,$$

$$R_{i0}^{k}=(\frac{\partial\varphi_{u}}{\partial x})_{i0}^{n+1}+\frac{v_{i1}^{n+1,k}-\varphi_{vi0}^{n+1}}{h_{y1}},1\le i\le N_{x}-1$$

Thanks to specific character of the problem computed are accordingly

$$R_{uij}^{k}=-\tau_{n+1}\frac{R_{i+1j}^{k}-R_{i-1j}^{k}}{h_{xi+1}+h_{xi}},$$

$$R_{vij}^{k}=-\tau_{n+1}\frac{R_{ij+1}^{k}-R_{ij-1}^{k}}{h_{yj+1}+h_{yj}},$$

$$i=1,\ldots,N_{x}-1,\ j=1,\ldots,N_{y}-1;$$

after which defined are

$$AR_{ij}^{k}=\frac{R_{ui+1j}^{k}-R_{ui-1j}^{k}}{h_{xi+1}+h_{xi}}+\frac{R_{vij+1}^{k}-R_{vij-1}^{k}}{h_{yj+1}+h_{yj}},$$

$$1\le i\le N_{x}-1,1\le j\le N_{y}-1,$$

$$AR_{N_x j}^k = \frac{-R_{uN_x-1j}^k}{h_{xN_x}}, 1 \le j \le N_y - 1,\ AR_{0j}^k = \frac{R_{u1j}^k}{h_{x1}}, 1 \le j \le N_y - 1,$$

$$AR_{iN_y}^k = \frac{-R_{viN_y-1}^k}{h_{yN_y}}, 1 \le i \le N_x - 1,\ AR_{i0}^k = \frac{R_{vi1}^k}{h_{y1}}, 1 \le i \le N_x - 1$$

Iteration parameter is calculated

$$\theta_k = -\sum_{i=0}^{N_x}\sum_{j=0}^{N_y} AR_{ij}^k * R_{ij}^k \Big/ \left(\sum_{i=0}^{N_x}\sum_{j=0}^{N_y} AR_{ij}^k\right)^2,$$

and further $p_{ij}^{k+1} = p_{ij}^k + \theta_k R_{ij}^k, i = 0,\ldots,N_x, j = 0,\ldots,N_y$

Iterations are stopped when inequalities are fulfilled

$$|R_{ij}^k| = \left|\left(u_{i+1j}^{n+1^k} - u_{i-1j}^{n+1^k}\right)\Big/(h_{xi+1} + h_{xi}) + \left(v_{ij+1}^{n+1^k} - v_{ij-1}^{n+1^k}\right)\Big/(h_{yj+1} + h_{yj})\right| \le \varepsilon,$$

$$1 \le i \le N_x - 1, 1 \le j \le N_y - 1$$

§6. *Program on TURBO-PASCAL*

Major stages of programming the scheme of paragraph ***[1].***

1 *. Array declaration:

$$u_{ij}^n \equiv UN[i,j],\ u_{ij}^{n+1} \equiv UN1[i,j],$$

$$v_{ij}^n \equiv VN[i,j],\ v_{ij}^{n+1} \equiv VN1[i,j],$$

$$p_{ij}^n \equiv PN[i,j],\ p_{ij}^{n+1} \equiv PN1[i,j],$$

$$p_{ij}^k \equiv PK[i,j],\ p_{ij}^{k+1} \equiv PK1[i,j],$$

$$h_{xi} \equiv HX[i],\ h_{yj} \equiv HY[j],\ A_{ij}^n \equiv AN[i,j].$$

2 *. Setting grid parameters N_x, N_y, h_{xi}, $i=1,\ldots,N_x$, h_{yj}, $j=1,\ldots,N_y$, τ_{n+1}, included into equations constants Re, k, b^0, L, H. For $Re>>1$ it is

necessary to apply a nonuniform grid: $h_{x1}=h_{x2}=1/Re^+$, $h_{x3}=2/Re^+$, $h_{xNx}=h_{xNx-1}=1/Re^+$, $h_{xNx-2}=2/Re^+$, $h_{xi}=(1-(h_{x1}+h_{x2}+h_{x3}+h_{xNx}+ +h_{xNx-1}+h_{xNx-2}))/(N_x-6)$, $4\le i\le N_x-3$; $h_{y1}=h_{y2}=1/Re^+$, $h_{y3}=2/Re^+$, $h_{yNt}=h_{yNy-1}=1/Re^+$, $h_{yNy-2}=2/Re^+$, $h_{yj}=(H/L-(h_{y1}+h_{y2}+h_{y3}+ +h_{yNy}+h_{yNy-1}+h_{yNy-2}))/(N_y-6)$, $4\le j\le N_y-3$, $\mathrm{Re}^+=\gamma\sqrt{\mathrm{Re}}, 0<\gamma<1$; when $Re<=500$ analytical grid $h_{xi}=1/N_x$, $i=1,...,N_x$, $h_{yj}=(H/L)/N_y$.

3*. Setting of initial conditions (25) in cycle by *i*,*j*.

***4**.** Setting in external (global) time cycle

$t_{n+1}=\sum_{m=0}^{n}\tau_{m+1}$ of boundary conditions (26),(27),(28),(29) and

computation in cycle by *i*,*j* $\overline{Q}^n_{uij}, \overline{Q}^n_{vij}, F^n_{uij}, F^n_{vij}$.

***5**.** Programming of iteration algorithm (71),(72),(73),(74),(75),(76),(77).

***6**.** Sending of calculated fields

$p_{ij}^{\bar{k}+1}=p_{ij}^{n+1}, u_{ij}^{n+1^{\bar{k}}}=u_{ij}^{n+1}, v_{ij}^{n+1^{\bar{k}}}=v_{ij}^{n+1}, \forall(i,j)$ into arrays

for $u_{ij}^n\equiv UN[i,j], v_{ij}^n\equiv VN[i,j], p_{ij}^n\equiv PN[i,j]$.

Global iteration program for calculation of longitudinal flow past a plate with asperity by vicious incompressible fluid when $H/L=8/\sqrt{\mathrm{Re}}$

```
PROGRAM  plastina;
USES crt; CONST NX=140; NY=40;nx1=40;nx2=50;ny1=10;
k=0;RE=40000      ; {in simulating turbulent conditions k#0, selected
by experimentation}
LABEL met, metk,met2,met4,met6,mettt,METPSI,MMM,SSS,TTT ;
VAR      out1, out2, out3: text;
            UN,UN1,UK,VN,VN1,VK,PK,PK1,PN,PN1,AN,FUN,
FVN,PSI:  array[0..NX,0..NY] of real;
HX: array[0..NX] of real; HY: array[0..NY] of real;
KJ,  i, j, KK, n, MK: integer; NN: real;
HK, AM,AX, EPSILON, T, QNUij, QNVij, TAY, S, TET,
```

```
 B0, HL, AMAX, UMAX, VMAX, PMAX, AU, AV, konu, konv,
unn, aunn, vnn, avnn: real;
begin
assign(out1,'ainur1.pas'); assign(out2,'ainur2.pas');
rewrite(out1); rewrite(out2);
        TET:=0.01;  B0:=0.1; HL:=8/SQRT(Re); EPSILON:=0.05;
NN:=1; MK:=5;
Writeln('hx[i] ');
for i:=1to NX do begin  HX[i]:=1./(NX-2*MK); Writeln('i= ', i,
HX[i]:9:4);
end;
Writeln('hy[j] ');
for j:=1 to NY do begin HY[j]:=HL/NY; Writeln('j= ', j, HY[j]:9:4);
end;
{2 comment: setting of initial conditions. In simulation of turbulent
conditions k#0 random disturbances are included into initial
conditions}
for i:=0 to NX do for j:=0 to NY do begin
UN[i,j]:=k*sin((i+j)*0.25)/10; VN[i,j]:=k*cos((i*j*3.46)/1000;
PN[i,j]:=0;
UN1[i,j]:= UN[i,j]; VN1[i,j]:= VN[i,j]; PN1[i,j[:=0; UK[i,j]:=0;
VK[i,j]:=0; PK[i,j[:=0; PK1[i,j]:=0;
end;       TAY:= 0.001;
n:=0; t:=0;
met: n:=n+1; t:=(n+1)*TAY;
for j:=0 to NY do begin UN1[0,j]:=exp(-B0/t)+k* sin(j*0.25)/40;
VN1[0,j]:=0; UK[0,j]:= UN1[0,j]; VK[0,j]:=0; VN[0,j]:=0end;
for i:=0 to NX do begin VN1[i,0]:=0; VK[i,0]:=0;end;
for i:=1 to NX-1 do for j:=1 to NY -1do begin
if(i>=nx1.and.i=<nx2.and.j=<ny1) goto MMM;unn:= UN[i,j];
vnn:=VN[i,j];aunn:=ABS(unn); avnn:=ABS(vnn);
konu:= ( aunn+unn)/2*( unn- UN[i-1,j])/ HX[i]+(unn-
aunn)/2*(UN[i+1,j]- unn)/HX[i+1]
+(avnn+vnn)/2*(unn- UN[i,j-1])/ Hy[j]+(vnn-avnn)/2*( UN[i,j+1]-
unn)/ Hy[j+1];
QNUij:=1/RE*(2/(HX[i+1]+HX[i])*((UN[i+1,j]- unn)/HX[i+1]-(
unn- UN[i-1,j])/ HX[i])+
```

```
2/(Hy[j+1]+Hy[j])*(( UN[i,j+1]-unn)/ Hy[j+1]-(unn- UN[i,j-1])/
Hy[j])-k*(( VN[i,j+1]-vnn)/ Hy[j+1]-(vnn- VN[i,j-1])/ Hy[j])/
(0.5*(Hy[j+1]+Hy[j]))) – konu;
konv:= ( aunn+unn)/2*( vnn- VN[i-1,j])/ HX[i]+(unn-
aunn)/2*(VN[i+1,j]- vnn)/HX[i+1]
+(avnn+vnn)/2*(vnn- VN[i,j-1])/ Hy[j]+(vnn-avnn)/2*( VN[i,j+1]-
vnn)/ Hy[j+1];
QNVij:=1/RE*(2/(HX[i+1]+HX[i])*((VN[i+1,j]- vnn)/HX[i+1]-(
vnn- VN[i-1,j])/ HX[i])+
2/(Hy[j+1]+Hy[j])*(( VN[i,j+1]-vnn)/ Hy[j+1]-(vnn- VN[i,j-1])/
Hy[j])-k*((UN[i+1,j]- unn)/HX[i+1]-( unn- UN[i-1,j])/ HX[i])/
(0.5*(HX[i+1]+HX[i])))-konv;
FUN[i,j]:=unn+TAY*QNUij;
FVN[i,j]:=vnn+TAY*QNVij;MMM:end;
if  n>1 then goto mettt;
for i:=1 to NX-1 do for j:=1 to NY -1do begin
AN[i,j]:=TAY/HX[i+1]*(1/ HX[i+1]+(1-(i-NX+1.5)/ABS(i-
NX+1.5))/(2*HX(i+1))+
TAY/HY[j+1]*(1/ HY[j+1]+(1-(j-NY+1.5)/ABS(j-
NY+1.5))/(2*HY(j+1));end;
mettt:
for i:=0 to NX do for j:=0 to NY do
 begin PK[i,j[:=PN[i,j[; end;KK:=0;metk:  KK:=KK+1
for j:=0 to NY do begin UK[0,j]:= UN1[0,j];  VK[0,j]:=0; end;
for i:=1 to NX-1 do for j:=1 to NY -1do begin
if(i>=nx1.and.i=<nx2.and.j=<ny1) goto SSS;UK[i,j]:= FUN[i,j]-
TAY*((PK[i,j]-PK[i-1,j])
/HX[i]-k*(PK[i,j]-PK[i,j-1])/HY[j]); VK[i,j]:= FVN[i,j]-TAY*(-
k*(PK[i,j]-PK[i-1,j])/HX[i]+(PK[i,j]-PK[i,j-1])/HY[j]); SSS:end;

AMAX:=0;
for i:=0 to NX-1 do for j:=0 to NY -1do begin
if(i>nx1.and.i=<nx2.and.j=<ny1) goto TTT;S:=(UK[i+1,j]-
UK[i,j])/HX[i+1]+ (VK[i,j+1]- VK[i,j])/HY[j+1]; PK1[i,j]:= PK[i,j]-
TET*S/(1+TET*AN[i,j]);
if ABS(S)>AMAX then
                                    AMAX:=ABS(S);TTT:end;
for j:=1 to NY do begin
```

UK[NX,j]:=2* UK[NX-1,j]- UK[NX-2,j];
VK[NX,j]:=2* VK[NX-1,j]- VK[NX-2,j];end;
for j:=0 to NY do begin
PK1[NX,j]:= 2*PK1[NX-1,j]- PK1[NX-2,j];end;
for i:=1 to NX do begin
if i<MK then UK[I,0]:=(4* UK[I,1]- UK[I,2])/3) ;
if i>(NX-MK) then UK[I,0]:=(4* UK[I,1]- UK[I,2])/3) ;
UK[I,NY]:=(4* UK[I,NY-1]- UK[I,NY-2])/3;
VK[I,NY]:=(4* VK[I,NY-1]- VK[I,NY-2])/3;End;
For i:=0 to NX do begin
PK1[I,NY]:= PK1[I,NY-1];end;
for i:=0 to NX do for j:=0 to NY do begin
PK[i,j]:= PK1[i,j]; UN1[i,j]:= UK[i,j];
VN1[i,j]:= VK[i,j]; end;
{16 comment: CHECK of fulfillment of continuity equation

AMAX= $\max\limits_{i,j} |\, (u_{i+1j}^{n+1^k} - u_{ij}^{n+1^k}) / h_{xi+1} + (v_{ij+1}^{n+1^k} - v_{ij}^{n+1^k}) / h_{yj+1} \,| \le \varepsilon$ }

If AMAX>EPSILON then goto metk;

UMAX:=0; PMAX:=0;
for i:=0 to NX do for j:=0 to NY do begin AU:=ABS(UN1[i,j]-UN[i,j]); AV:=ABS(VN1[i,j]-VN[i,j]);

UMAX:= UMAX+AU+AV;
PN[i,j]:= PK1[i,j]; if ABS(PK1[i,j])>PMAX then PMAX := ABS(PK1[i,j]); UN[i,j]:= UN1[i,j];VN[i,j]:= VN1[i,j]; end;

UMAX:= UMAX/(NX*NY); writeln (out2, ‘number of time layer

N=’,N:9:0, ‘ AMAX=’ , AMAX, ‘KK=’ , KK);

if N<500 then goto met; if N=1500 then goto met6;

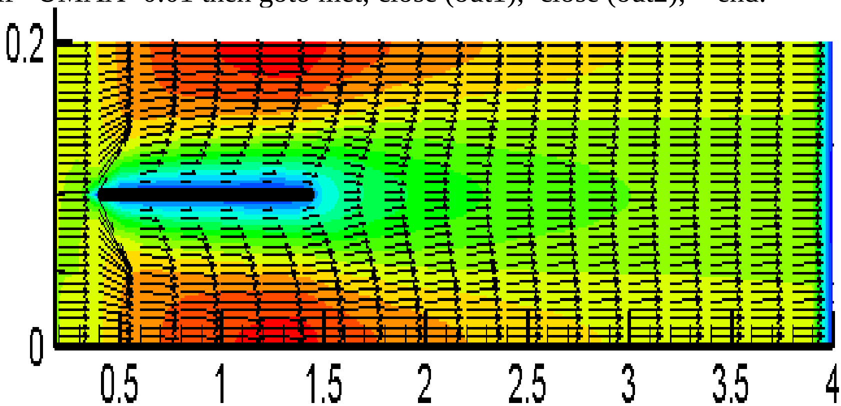
met4: NN:=NN+1; {17 comment: Ptintout each 1000 time layers}

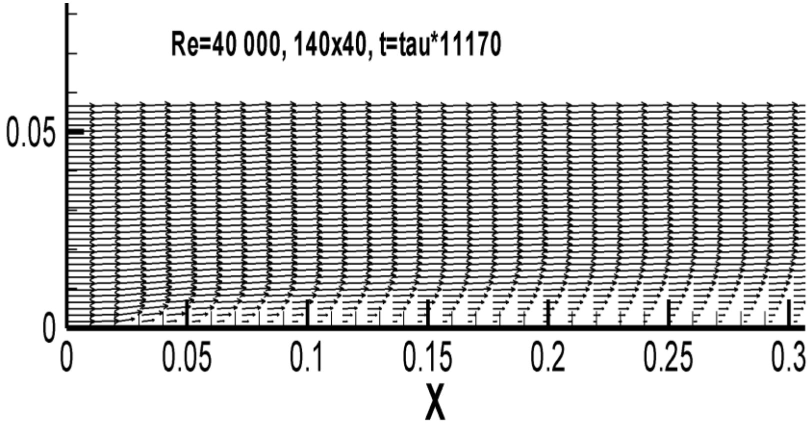
if NN<>1000 then goto met2; met6: writeln (out1, ‘number of time

layer N=’,N:9:0, ‘ AMAX=’ ,

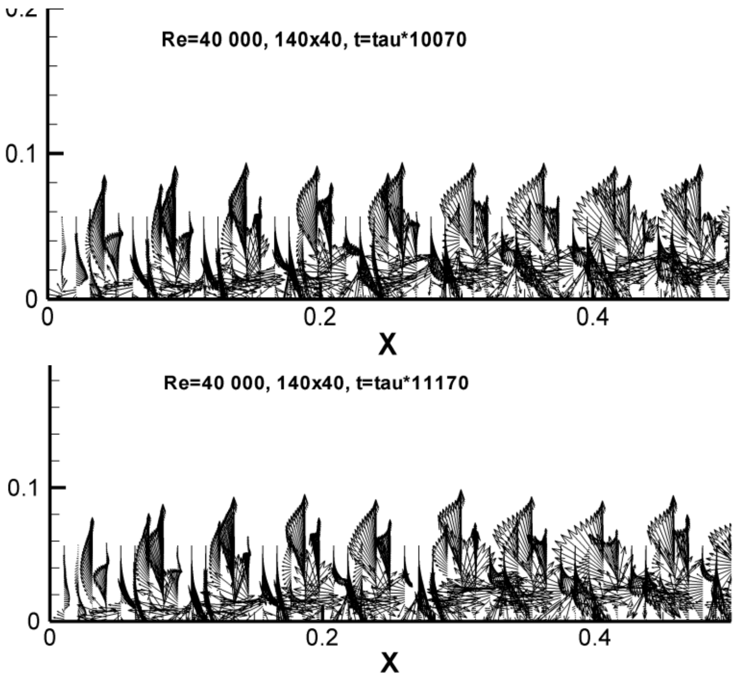
```
AMAX, ‘KK=’ , KK );writeln (out2, ‘number of time layer
N=’,N:9:0, ‘ AMAX=’ , AMAX, ‘KK=’ , KK );
writeln (‘ N= ‘, N:9:0);
 writeln ( ‘Residual=’ ,  UMAX, ‘ AMAX=’ , AMAX, ‘KK=’ , KK );
 writeln (out1, ‘UN=’); writeln (out2, ‘PN=’);
for KJ:=0 to NY do begin j:=NY-KJ; write (out1, ‘j=’ , j);
for i:=0 to NX do
                    write (out1, ‘   ‘, UN[i,j]:9:4); writeln (out1); end;
writeln (out1, ‘*********************’); writeln (out1, ‘VN=’);

for KJ:=0 to NY do begin j:=NY-KJ; write (out1, ‘j=’ , j);
for i:=0 to NX do
                    write (out1, ‘   ‘, VN[i,j]:9:4);writeln (out1); end;
writeln (out2, ‘PN=’);
for KJ:=0 to NY do begin j:=NY-KJ; write (out2, ‘j=’ , j);
for i:=0 to NX do begin PK1[i,j]:= PK1[i,j]/PMAX;
write (out2, ‘   ‘, PK1[i,j]:9:4);writeln (out2);
end;end;  met2:  if  NN=1000 then  NN:=0;
if   UMAX>0.01 then goto met; close (out1);  close (out2);    end.
```





Field of averaged velocity vector $\vec{\bar{v}}$ in longitudinal flow past plate.



Fields of pulsating velocity vector $\vec{v}\,'$ in longitudinal flow past plate at various moments of time.

§ 7. Approximation of convection members without "schematic viscosity"

Universal is multipoint approximation of a convection member, not containing "schematic viscosity", (ref. *Jakupov K.B.* /3/), idea of which is as follows.

From expansion into *Taylor* series

$$\Phi_{i-1j}^{n} = \Phi_{ij}^{n} - h_{xi} \left(\frac{\partial \Phi}{\partial x}\right)_{ij}^{n} + \frac{h_{xi}^{2}}{2} \left(\frac{\partial^{2} \Phi}{\partial x^{2}}\right)_{ij}^{n} + O(h_{xi}^{2})$$

results

$$\left(\frac{\partial \Phi}{\partial x}\right)_{ij}^{n} = \frac{\Phi_{ij}^{n} - \Phi_{i-1j}^{n}}{h_{xi}} + \frac{h_{xi}}{2} \left(\frac{\partial^{2} \Phi}{\partial x^{2}}\right)_{ij}^{n} + O(h_{xi}^{2})$$

Standing here 2[nd] derivative is approximated as follows: in near-boundary node $i = 1$ by formula

$$\left(\frac{\partial^{2} \Phi}{\partial x^{2}}\right)_{ij}^{n} = \left(\frac{\Phi_{i+2j}^{n} - \Phi_{i+1j}^{n}}{h_{xi+2}} - \frac{\Phi_{i+1j}^{n} - \Phi_{ij}^{n}}{h_{xi+1}}\right) \frac{2}{h_{xi+2} + h_{xi+1}} + O(h_{xi+2} + h_{xi+1}),$$

In the rest of nodes $i = 2, \ldots, N_x - 1$ by another formula

$$\left(\frac{\partial^{2} \Phi}{\partial x^{2}}\right)_{ij}^{n} = \left(\frac{\Phi_{ij}^{n} - \Phi_{i-1j}^{n}}{h_{xi}} - \frac{\Phi_{i-1j}^{n} - \Phi_{i-2j}^{n}}{h_{xi-1}}\right) \frac{2}{h_{xi} + h_{xi-1}} + O(h_{xi-1} + h_{xi})$$

These approximations are applied in case of nonnegative coefficient $u_{ij}^{n} \geq 0$.

From similar expansion into *Taylor* series

$$\Phi_{i+1j}^{n} = \Phi_{ij}^{n} + h_{xi+1} \left(\frac{\partial \Phi}{\partial x}\right)_{ij}^{n} + \frac{h_{xi+1}^{2}}{2} \left(\frac{\partial^{2} \Phi}{\partial x^{2}}\right)_{ij}^{n} + O(h_{xi+1}^{2})$$

derived is

$$\left(\frac{\partial \Phi}{\partial x}\right)_{ij}^{n} = \frac{\Phi_{i+1j}^{n} - \Phi_{ij}^{n}}{h_{xi+1}} - \frac{h_{xi+1}}{2} \left(\frac{\partial^{2} \Phi}{\partial x^{2}}\right)_{ij}^{n} + O(h_{xi+1}^{2})$$

Standing here 2nd derivative is approximated as follows: in near-boundary nodes $i = 1, \ldots, N_x - 2$ by formula

$$(\frac{\partial^2\Phi}{\partial x^2})^n_{ij}=(\frac{\Phi^n_{i+2j}-\Phi^n_{i+1j}}{h_{xi+2}}-\frac{\Phi^n_{i+1j}-\Phi^n_{ij}}{h_{xi+1}})\frac{2}{h_{xi+2}+h_{xi+1}}+O(h_{xi+2}+h_{xi+1}),$$

in near-boundary node $i=N_x-1$ by another formula

$$(\frac{\partial^2\Phi}{\partial x^2})^n_{ij}=(\frac{\Phi^n_{ij}-\Phi^n_{i-1j}}{h_{xi}}-\frac{\Phi^n_{i-1j}-\Phi^n_{i-2j}}{h_{xi-1}})\frac{2}{h_{xi}+h_{xi-1}}+O(h_{xi-1}+h_{xi})$$

These approximations are applied in case of nonpositive coefficient $u^n_{ij}\le 0$. Discrete function *sign:*

$$for\ A<0, signA=-1,\ for\ A=0, signA=0,\ for\ A>0, signA=+1$$

allows writing approximations not containing "schematic viscosity" in the form of a uniform formula

$$\begin{aligned}
u^n_{ij}(\frac{\partial\Phi}{\partial x})^n_{ij} \ &= \ \frac{|u^n_{ij}|+u^n_{ij}}{2}\{\frac{\Phi^n_{ij}-\Phi^n_{i-1j}}{h_{xi}}+\\
&+\frac{h_{xi}}{2}[(\frac{\Phi^n_{ij}-\Phi^n_{i-1j}}{h_{xi}}-\frac{\Phi^n_{i-1j}-\Phi^n_{i-2j}}{h_{xi-1}})\frac{2}{h_{xi}+h_{xi-1}}\frac{1+sign(i-1.5)}{2}+\\
&+(\frac{\Phi^n_{i+2j}-\Phi^n_{i+1j}}{h_{xi+2}}-\frac{\Phi^n_{i+1j}-\Phi^n_{ij}}{h_{xi+1}})\frac{2}{h_{xi+2}+h_{xi+1}}\frac{1-sign(i-1.5)}{2}]\}+\\
&+\frac{u^n_{ij}-|u^n_{ij}|}{2}\{\frac{\Phi^n_{i+1j}-\Phi^n_{ij}}{h_{xi+1}}-\\
&-\frac{h_{xi+1}}{2}[(\frac{\Phi^n_{i+2j}-\Phi^n_{i+1j}}{h_{xi+2}}-\frac{\Phi^n_{i+1j}-\Phi^n_{ij}}{h_{xi+1}})\frac{2}{h_{xi+2}+h_{xi+1}}\frac{1-sign(i-N_x+1.5)}{2}+\\
&+(\frac{\Phi^n_{ij}-\Phi^n_{i-1j}}{h_{xi}}-\frac{\Phi^n_{i-1j}-\Phi^n_{i-2j}}{h_{xi-1}})\frac{2}{h_{xi}+h_{xi-1}}\frac{1+sign(i-N_x+1.5)}{2}]\}+O(\hbar^2_{xi})\equiv\\
&\equiv\frac{|u^n_{ij}|+u^n_{ij}}{2}\Phi^n_{\bar{x}^0}+\frac{u^n_{ij}-|u^n_{ij}|}{2}\Phi^n_{x^0}+O(\hbar^2_{xi})
\end{aligned} \tag{73}$$

On analytical grid $h_{xi+1}=h_{xi}=h_x$ approximation is simplified:

$$\Phi^n_{\bar{x}^0} \equiv \frac{3\Phi^n_{ij} - 4\Phi^n_{i-1j} + \Phi^n_{i-2j}}{2h_x} * \frac{1+sign(i-1{,}5)}{2} +$$
$$+\frac{3\Phi^n_{ij} - 2\Phi^n_{i-1j} - 2\Phi^n_{i+1j} + \Phi^n_{i+2j}}{2h_x} * \frac{1-sign(i-1{,}5)}{2},$$
$$\Phi^n_{x^0} \equiv \frac{4\Phi^n_{i+1j} - 3\Phi^n_{ij} - \Phi^n_{i+2j}}{2h_x} * \frac{1-sign(i-N_x+1{,}5)}{2} +$$
$$+\frac{2\Phi^n_{i+1j} + 2\Phi^n_{i-1j} - 3\Phi^n_{ij} - \Phi^n_{i-2j}}{2h_x} * \frac{1+sign(i-N_x+1{,}5)}{2}\}$$

Similar approximation of 2nd order of accuracy is applied also for other convection members of $v^n_{ij}(\frac{\partial\Phi}{\partial y})^n_{ij}$ type.

Conditions of convergence and stability of explicit scheme impose constraints on time steps as follows

$$[1-\tau_{n+1}(\frac{2}{\operatorname{Re} h_x^2} + \frac{3|u^n_{ji}|}{2h_x} + \frac{2}{\operatorname{Re} h_y^2} + \frac{3|v^n_{ji}|}{2h_y})] \geq 0,$$
$$i = 1,\ldots,N_x - 1,\ j = 1,\ldots,N_y - 1,\ \ n = 0,1,2,\ldots,N_\tau - 1,$$
$$h_x = \min_i h_{xi}, h_y = \min_j h_{yj}.$$

In algorithms of realizations of the above-given semi-implicit schemes in case of approximations of convection members without «schematic viscosity» changed will be only content Q^n_{uij}, Q^n_{vij}, included into F^n_{uij}, F^n_{vij}. They will be represented with account to short representation in the form:

$$Q^n_{uij} = \frac{1}{\operatorname{Re}} [\frac{2}{h_{xi+1} + h_{xi}}\left(\frac{u^n_{i+1j} - u^n_{ij}}{h_{xi+1}} - \frac{u^n_{ij} - u^n_{i-1j}}{h_{xi}}\right) + \frac{2}{h_{yj+1} + h_{yj}} *$$

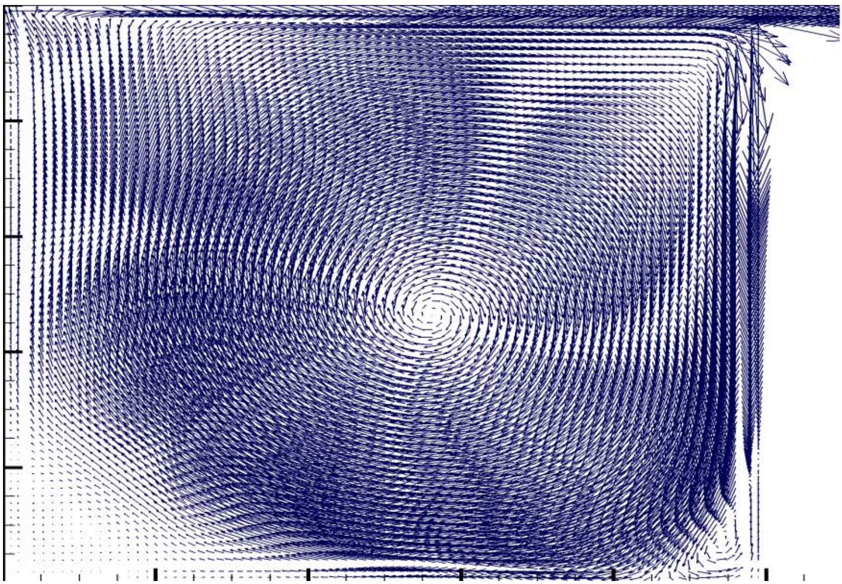
$$*\left(\frac{u^n_{ij+1}-u^n_{ij}}{h_{yj+1}}-\frac{u^n_{ij}-u^n_{ij-1}}{h_{yj}}\right)\Bigg]$$

$$-\{\frac{|u^n_{ij}|+u^n_{ij}}{2}u^n_{\bar{x}^0}+\frac{u^n_{ij}-|u^n_{ij}|}{2}u^n_{x^0}+\frac{|v^n_{ij}|+v^n_{ij}}{2}u^n_{\bar{y}^0}+\frac{v^n_{ij}-|v^n_{ij}|}{2}u^n_{y^0}\},$$

$$Q^n_{vij}=\frac{1}{\mathrm{Re}}\Bigg[\frac{2}{h_{xi+1}+h_{xi}}\left(\frac{v^n_{i+1j}-v^n_{ij}}{h_{xi+1}}-\frac{v^n_{ij}-v^n_{i-1j}}{h_{xi}}\right)+\frac{2}{h_{yj+1}+h_{yj}}*$$

$$*\left(\frac{v^n_{ij+1}-v^n_{ij}}{h_{yj+1}}-\frac{v^n_{ij}-v^n_{ij-1}}{h_{yj}}\right)\Bigg]$$

$$-\{\frac{|u^n_{ij}|+u^n_{ij}}{2}v^n_{\bar{x}^0}+\frac{u^n_{ij}-|u^n_{ij}|}{2}v^n_{x^0}+\frac{|v^n_{ij}|+v^n_{ij}}{2}v^n_{\bar{y}^0}+\frac{v^n_{ij}-|v^n_{ij}|}{2}v^n_{y^0}\}$$



Established field of velocity vector in cavity with the top moving cover. Grid 140x80, Re=500.

§ 8. Semi-implicit schemes for solving of three-dimensional equations of vicious incompressible fluid

For three-dimensional equations, suggested in /2/ for simulating of both, laminar ($k=0$) as well as turbulent flows ($k\neq 0$) of incompressible fluid

$$\rho\frac{du}{dt}+\frac{\partial p}{\partial x}=\rho F_x+\frac{\partial}{\partial x}(\mu\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})+\frac{\partial}{\partial z}(\mu\frac{\partial u}{\partial z})-$$

$$-k\frac{\partial}{\partial y}[u(u^2+w^2)^{-\frac{1}{2}}(p-\mu\frac{\partial v}{\partial y})]-k\frac{\partial}{\partial z}[u(u^2+v^2)^{-\frac{1}{2}}(p-\mu\frac{\partial w}{\partial z})],$$

$$\rho\frac{dv}{dt}+\frac{\partial p}{\partial y}=\rho F_y+\frac{\partial}{\partial x}(\mu\frac{\partial v}{\partial x})+\frac{\partial}{\partial y}(\mu\frac{\partial v}{\partial y})+\frac{\partial}{\partial z}(\mu\frac{\partial v}{\partial z})-$$

$$-k\frac{\partial}{\partial x}[v(v^2+w^2)^{-\frac{1}{2}}(p-\mu\frac{\partial u}{\partial x})]-k\frac{\partial}{\partial z}[v(v^2+v^2)^{-\frac{1}{2}}(p-\mu\frac{\partial w}{\partial z})],$$

$$\rho\frac{dw}{dt}+\frac{\partial p}{\partial z}=\rho F_z+\frac{\partial}{\partial x}(\mu\frac{\partial w}{\partial x})+\frac{\partial}{\partial y}(\mu\frac{\partial w}{\partial y})+\frac{\partial}{\partial z}(\mu\frac{\partial w}{\partial z})-$$

$$-k\frac{\partial}{\partial x}[w(w^2+w^2)^{-\frac{1}{2}}(p-\mu\frac{\partial u}{\partial x})]-k\frac{\partial}{\partial y}[w(w^2+w^2)^{-\frac{1}{2}}(p-\mu\frac{\partial v}{\partial y})],$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0,$$

$\frac{d}{dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}$, with limiting conditions

$$u|_{t=0}=d_u(x,y,z), v|_{t=0}=d_v(x,y,z), w|_{t=0}=\varphi_w(x,y,z),$$

$$u|_{\sigma}=\varphi_u(x,y,z,t), v|_{\sigma}=\varphi_v(x,y,z,t), w|_{\sigma}=\varphi_w(x,y,z,t)$$

Schemes and methods of global iterations given above are easily generalized. By analogy with directions x,y let's introduce nodes on z axis:

$$z_m, m=0,1,..N_z, h_{zm}=z_m-z_{m-1}, m=1,...,N_z, f(x_i,y_j,z_m,t_n)\equiv f^n_{ijm},$$

$$\overline{\Omega}_h = \{x_i, i = 0,1,\ldots,N_x, y_j, j = 0,1,\ldots,N_y, z_m, m = 0,1,\ldots N_z\},$$

$$\Omega_h = \{x_i, i = 1,\ldots,N_x - 1, y_j, j = 1,\ldots,N_y - 1, z_m, m = 1,\ldots N_z - 1\}$$

Let convection members in the general case be approximated by universal formula of (73) type, i.e. without “schematic viscosity” and let’s take a two-dimensional scheme (57)-(60) as a sample. Let’s introduce short notation of coefficient when p_{ijm}^{n+1} in scheme with central differences for pressure gradient and continuity equation:

$$A_{ijm} = \tau_{n+1}\{\frac{1}{h_{xi+1} + h_{xi}}[\frac{1}{h_{xi+2} + h_{xi+1}}\frac{1 - sign(i - N_x + 1.5)}{2} +$$

$$+\frac{1}{h_{xi} + h_{xi-1}}\frac{1 + sign(i - 1.5)}{2}] +$$

$$+\frac{1}{h_{yj+1} + h_{yj}}[\frac{1}{h_{yj+2} + h_{yj+1}}\frac{1 - sign(j - N_y + 1.5)}{2} +$$

$$+\frac{1}{h_{yj} + h_{yj-1}}\frac{1 + sign(j - 1.5)}{2}] +$$

$$+\frac{1}{h_{zm+1} + h_{zm}}[\frac{1}{h_{zm+2} + h_{zm+1}}\frac{1 - sign(m - N_z + 1.5)}{2} +$$

$$+\frac{1}{h_{zm} + h_{zm-1}}\frac{1 + sign(m - 1.5)}{2}]\},$$

$$i = 1,\ldots,N_x - 1,\ j = 1,\ldots,N_y - 1, m = 1,\ldots,N_z - 1;$$

$$A_{N_x jm} = \tau_{n+1}\frac{1}{h_{xN_x}(h_{xN_x} + h_{xN_x - 1})},$$

$$A_{0jm}=\tau_{n+1}\frac{1}{h_{x1}(h_{x2}+h_{x1})},\ j=1,\ldots,N_y-1, m=1,\ldots,N_z-1;$$

$$A_{iN_ym}=\tau_{n+1}\frac{1}{h_{yN_y}(h_{yN_y}+h_{yN_y-1})},$$

$$A_{i0m}=\tau_{n+1}\frac{1}{h_{y1}(h_{y2}+h_{y1})},i=1,\ldots,N_x-1, m=1,\ldots,N_z-1,$$

$$A_{ijN_z}=\tau_{n+1}\frac{1}{h_{zN_z}(h_{zN_z}+h_{zN_z-1})},$$

$$A_{ij0}=\tau_{n+1}\frac{1}{h_{z1}(h_{z2}+h_{z1})},i=1,\ldots,N_x-1,\ j=1,\ldots,N_y-1,$$

The global iteration process is built as follows:

$$u_{ijm}^{n+1^S}=F_{uijm}^{n}-\tau_{n+1}\{\frac{p_{i+1jm}^{s}-p_{i-1jm}^{s}}{h_{xi+1}+h_{xi}}+\frac{k}{h_{yj+1}+h_{yj}}[u_{ij+1m}^{n}(u_{ij+1m}^{n2}+$$
$$+w_{ij+1m}^{n2})^{-0,5}p_{ij+1m}^{s}-u_{ij-1m}^{n}(u_{ij-1m}^{n2}+w_{ij-1m}^{n2})^{-0,5}p_{ij-1m}^{s}]+$$
$$+k[u_{ijm+1}^{n}(u_{ijm+1}^{n2}+v_{ijm+1}^{n2})^{-0,5}p_{ijm+1}^{s}-$$
$$-u_{ijm-1}^{n}(u_{ijm-1}^{n2}+v_{ijm-1}^{n2})^{-0,5}p_{ijm-1}^{s}]/(h_{zm+1}+h_{zm})\},\quad (74)$$

$$v_{ijm}^{n+1^S}=F_{vijm}^{n}-\tau_{n+1}\{\frac{p_{ij+1m}^{s}-p_{ij-1m}^{s}}{h_{yj+1}+h_{yj}}+\frac{k}{h_{xi+1}+h_{xi}}[v_{i+1jm}^{n}(v_{i+1jm}^{n2}+$$
$$+w_{i+1jm}^{n2})^{-0,5}p_{i+1jm}^{s}-v_{i-1jm}^{n}(v_{i-1jm}^{n2}+w_{i-1jm}^{n2})^{-0,5}p_{i-1jm}^{s}]+$$
$$+k[v_{ijm+1}^{n}(u_{ijm+1}^{n2}+v_{ijm+1}^{n2})^{-0,5}p_{ijm+1}^{s}-$$
$$-v_{ijm-1}^{n}(u_{ijm-1}^{n2}+v_{ijm-1}^{n2})^{-0,5}p_{ijm-1}^{s}]/(h_{zm+1}+h_{zm})\},\quad (75)$$

$$w_{ijm}^{n+1^S}=F_{wijm}^{n}-\tau_{n+1}\{\frac{p_{ijm+1}^{s}-p_{ijm-1}^{s}}{h_{zm+1}+h_{zm}}+\frac{k}{h_{xi+1}+h_{xi}}[w_{i+1jm}^{n}(v_{i+1jm}^{n2}+$$

$$+w_{i+1jm}^{n2})^{-0,5}p_{i+1jm}^{s}-w_{i-1jm}^{n}(v_{i-1jm}^{n2}+w_{i-1jm}^{n2})^{-0,5}p_{i-1jm}^{s}]+$$

$$+k[w_{ij+1m}^{n}(u_{ij+1m}^{n2}+w_{ij+1m}^{n2})^{-0,5}p_{ij+1m}^{s}-$$

$$-w_{ij-1m}^{n}(u_{ij-1m}^{n2}+w_{ij-1m}^{n2})^{-0,5}p_{ij-1m}^{s}]/(h_{yj+1}+h_{yj})\}, \qquad (76)$$

$$i=1,\ldots,N_x-1,\ j=1,\ldots,N_y-1,\ m=1,\ldots,N_z-1;$$

$$(1+A_{ijm}\theta)\frac{p_{ijm}^{s+1}-p_{ijm}^{s}}{\theta}+\frac{u_{i+1jm}^{n+1^S}-u_{i-1jm}^{n+1^S}}{h_{xi+1}+h_{xi}}+$$

$$+\frac{v_{ij+1m}^{n+1^S}-v_{ij-1m}^{n+1^S}}{h_{yj+1}+h_{yj}}+\frac{w_{ijm+1}^{n+1^S}-w_{ijm-1}^{n+1^S}}{h_{zm+1}+h_{zm}}=0,$$

$$i=1,\ldots,N_x-1,\ j=1,\ldots,N_y-1,\ m=1,\ldots,N_z-1;$$

$$(1+A_{N_xjm}\theta)\frac{p_{N_xjm}^{s+1}-p_{N_xjm}^{s}}{\theta}+\frac{\varphi_{uN_xjm}^{n+1}-u_{N_x-1jm}^{n+1^S}}{h_{xN_x}}+(\frac{\partial\varphi_v}{\partial y})_{N_xjm}^{n+1}+(\frac{\partial\varphi_z}{\partial z})_{N_xjm}^{n+1}=0,$$

$$(1+A_{0jm}\theta)\frac{p_{0jm}^{s+1}-p_{0jm}^{s}}{\theta}+\frac{u_{1jm}^{n+1^S}-\varphi_{u0jm}^{n+1}}{h_{x1}}+(\frac{\partial\varphi_v}{\partial y})_{0jm}^{n+1}+(\frac{\partial\varphi_z}{\partial z})_{0jm}^{n+1}=0,$$

$$1\le j\le N_y-1, 1\le m\le N_z-1;$$

$$(1+A_{iN_ym}\theta)\frac{p_{iN_ym}^{s+1}-p_{iN_ym}^{s}}{\theta}+\frac{\varphi_{viN_ym}^{n+1}-v_{iN_y-1m}^{n+1^S}}{h_{yN_y}}+(\frac{\partial\varphi_u}{\partial x})_{iN_ym}^{n+1}+(\frac{\partial\varphi_w}{\partial z})_{iN_ym}^{n+1}=0,$$

$$(1+A_{i0m}\theta)\frac{p_{i0m}^{s+1}-p_{i0m}^{s}}{\theta}+\frac{v_{i1m}^{n+1^S}-\varphi_{vi0m}^{n+1}}{h_{y1}}+(\frac{\partial\varphi_u}{\partial x})_{i0m}^{n+1}+(\frac{\partial\varphi_w}{\partial z})_{i0m}^{n+1}=0,$$

$$1\le i\le N_x-1, 1\le m\le N_z-1;$$

$$(1+A_{ijN_z}\theta)\frac{p_{ijN_z}^{s+1}-p_{ijN_z}^{s}}{\theta}+\frac{\varphi_{wijN_z}^{n+1}-w_{ijN_z-1}^{n+1^S}}{h_{zN_z}}+(\frac{\partial\varphi_u}{\partial x})_{ijN_z}^{n+1}+(\frac{\partial\varphi_v}{\partial y})_{ijN_z}^{n+1}=0,$$

$$(1+A_{ij0}\theta)\frac{p_{ij0}^{s+1}-p_{ij0}^{s}}{\theta}+\frac{w_{ij1}^{n+1^S}-\varphi_{wij0}^{n+1}}{h_{z1}}+(\frac{\partial\varphi_u}{\partial x})_{ij0}^{n+1}+(\frac{\partial\varphi_v}{\partial y})_{ij0}^{n+1}=0,$$

$$1\le i\le N_x-1, 1\le j\le N_y-1$$

***Global iterations of finding pressure by Krasnoselsky - Krein method of minimal residuals (*m.m.r.)**

By equations (74) - (75) - (76) computed are three components $u_{ijm}^{n+1^S}, v_{ijm}^{n+1^S}, w_{ijm}^{n+1^S}$, after which, residuals of *s*-iteration

$$R_{ijm}^{s}=\frac{u_{i+1jm}^{n+1^S}-u_{i-1jm}^{n+1^S}}{h_{xi+1}+h_{xi}}+\frac{v_{ij+1m}^{n+1^S}-v_{ij-1m}^{n+1^S}}{h_{yj+1}+h_{yj}}+\frac{w_{ijm+1}^{n+1^S}-w_{ijm-1}^{n+1^S}}{h_{zm+1}+h_{zm}},$$

$$i=1,\dots,N_x-1,\ j=1,\dots,N_y-1, m=1,\dots,N_z-1;$$

$$R_{N_xjm}^{s}=\frac{\varphi_{uN_xjm}^{n+1}-u_{N_x-1jm}^{n+1^S}}{h_{xN_x}}+(\frac{\partial\varphi_v}{\partial y})_{N_xjm}^{n+1}+(\frac{\partial\varphi_z}{\partial z})_{N_xjm}^{n+1},$$

$$R_{0jm}^{s}=\frac{u_{1jm}^{n+1^S}-\varphi_{u0jm}^{n+1}}{h_{x1}}+(\frac{\partial\varphi_v}{\partial y})_{0jm}^{n+1}+(\frac{\partial\varphi_z}{\partial z})_{0jm}^{n+1},$$

$$1\le j\le N_y-1, 1\le m\le N_z-1;$$

$$R_{iN_ym}^{s}=\frac{\varphi_{viN_ym}^{n+1}-v_{iN_y-1m}^{n+1^S}}{h_{yN_y}}+(\frac{\partial\varphi_u}{\partial x})_{iN_ym}^{n+1}+(\frac{\partial\varphi_w}{\partial z})_{iN_ym}^{n+1}=0,$$

$$R^s_{i0m} = \frac{v^{n+1^S}_{i1m} - \varphi^{n+1}_{vi0m}}{h_{y1}} + (\frac{\partial \varphi_u}{\partial x})^{n+1}_{i0m} + (\frac{\partial \varphi_w}{\partial z})^{n+1}_{i0m} = 0,$$

$$1 \le i \le N_x - 1, 1 \le m \le N_z - 1;$$

$$R^s_{ijN_z} = \frac{\varphi^{n+1}_{wijN_z} - w^{n+1^S}_{ijN_z-1}}{h_{zN_z}} + (\frac{\partial \varphi_u}{\partial x})^{n+1}_{ijN_z} + (\frac{\partial \varphi_v}{\partial y})^{n+1}_{ijN_z},$$

$$R^s_{ij0} = \frac{w^{n+1^S}_{ij1} - \varphi^{n+1}_{wij0}}{h_{z1}} + (\frac{\partial \varphi_u}{\partial x})^{n+1}_{ij0} + (\frac{\partial \varphi_v}{\partial y})^{n+1}_{ij0},$$

$$\mathbf{1 \le i \le N_x - 1, 1 \le j \le N_y - 1}$$

Thanks to specific nature of the problem computed are accordingly

$$R^s_{uijm} = -\tau_{n+1}\{\frac{R^s_{i+1jm} - R^s_{i-1jm}}{h_{xi+1} + h_{xi}} + \frac{k}{h_{yj+1} + h_{yj}}[u^n_{ij+1m}(u^{n2}_{ij+1m} +$$

$$+ w^{n2}_{ij+1m})^{-0,5} R^s_{ij+1m} - u^n_{ij-1m}(u^{n2}_{ij-1m} + w^{n2}_{ij-1m})^{-0,5} R^s_{ij-1m}] +$$

$$+ \frac{k}{h_{zm+1} + h_{zm}}[u^n_{ijm+1}(u^{n2}_{ijm+1} + v^{n2}_{ijm+1})^{-0,5} R^s_{ijm+1} -$$

$$- u^n_{ijm-1}(u^{n2}_{ijm-1} + v^{n2}_{ijm-1})^{-0,5} R^s_{ijm-1}]\},$$

$$R^s_{vijm} = -\tau_{n+1}\{\frac{R^s_{ij+1m} - R^s_{ij-1m}}{h_{yj+1} + h_{yj}} + \frac{k}{h_{xi+1} + h_{xi}}[v^n_{i+1jm}(v^{n2}_{i+1jm} +$$

$$+ w^{n2}_{i+1jm})^{-0,5} R^s_{i+1jm} - v^n_{i-1jm}(v^{n2}_{i-1jm} + w^{n2}_{i-1jm})^{-0,5} R^s_{i-1jm}] +$$

$$+ \frac{k}{h_{zm+1} + h_{zm}}[v^n_{ijm+1}(u^{n2}_{ijm+1} + v^{n2}_{ijm+1})^{-0,5} R^s_{ijm+1} -$$

$$-v^{n}_{ijm-1}(u^{n2}_{ijm-1}+v^{n2}_{ijm-1})^{-0,5}R^{s}_{ijm-1}]\},$$

$$R^{s}_{wijm}=-\tau_{n+1}\{\frac{R^{s}_{ijm+1}-R^{s}_{ijm-1}}{h_{zm+1}+h_{zm}}+\frac{k}{h_{xi+1}+h_{xi}}[w^{n}_{i+1jm}(v^{n2}_{i+1jm}+$$

$$+w^{n2}_{i+1jm})^{-0,5}R^{s}_{i+1jm}-w^{n}_{i-1jm}(v^{n2}_{i-1jm}+w^{n2}_{i-1jm})^{-0,5}R^{s}_{i-1jm}]+$$

$$+\frac{k}{h_{yj+1}+h_{yj}}[w^{n}_{ij+1m}(u^{n2}_{ij+1m}+w^{n2}_{ij+1m})^{-0,5}R^{s}_{ij+1m}-$$

$$-w^{n}_{ij-1m}(u^{n2}_{ij-1m}+w^{n2}_{ij-1m})^{-0,5}R^{s}_{ij-1m}]\},$$

$$i=1,\dots,N_x-1,\ j=1,\dots,N_y-1,m=1,\dots,N_z-1;$$

after which defined are

$$AR^{s}_{ijm}=\frac{R^{s}_{ui+1jm}-R^{s}_{ui-1jm}}{h_{xi+1}+h_{xi}}+\frac{R^{s}_{vij+1m}-R^{s}_{vij-1m}}{h_{yj+1}+h_{yj}}+\frac{R^{s}_{wijm+1}-R^{s}_{vijm-1}}{h_{zm+1}+h_{zm}},$$

$$i=1,\dots,N_x-1,\ j=1,\dots,N_y-1,m=1,\dots,N_z-1;$$

$$AR^{s}_{N_xjm}=\frac{-R^{s}_{uN_x-1jm}}{h_{xN_x}},AR^{s}_{0jm}=\frac{R^{s}_{u1jm}}{h_{x1}},1\le j\le N_y-1,1\le m\le N_m-1;$$

$$AR^{s}_{i0m}=\frac{R^{s}_{vi1m}}{h_{y1}},AR^{s}_{iN_ym}=\frac{-R^{s}_{viN_y-1m}}{h_{yN_y}},1\le i\le N_x-1,1\le m\le N_m-1,$$

$$AR^{s}_{ij0}=\frac{R^{s}_{wij1}}{h_{z1}},AR^{s}_{ijN_z}=\frac{-R^{s}_{wijN_z-1}}{h_{zN_z}},1\le i\le N_x-1,1\le j\le N_y-1$$

Iteration parameter is calculated

$$\theta_s=-\sum_{i=0}^{N_x}\sum_{j=0}^{N_y}\sum_{m=0}^{N_z}AR^{s}_{ijm}*R^{s}_{ijm}\Bigg/(\sum_{i=0}^{N_x}\sum_{j=0}^{N_y}\sum_{m=0}^{N_z}AR^{s}_{ijm})^2,$$

and further $p^{s+1}_{ijm}=p^{s}_{ijm}+\theta_sR^{s}_{ijm},i=0,\dots,N_x,j=0,\dots,N_y,k=0,\dots,N_z$

Iterations are stopped when inequalities are fulfilled

$$| R_{ijm}^{s^*} | \leq \varepsilon, i = 0,\ldots,N_x,\ j = 0,\ldots,N_y, m = 0,\ldots,N_z$$

Last approximations $p_{ijm}^{s^*}, u_{ijm}^{n+1^{s^*}}, v_{ijm}^{n+1^{s^*}}, w_{ijm}^{n+1^{s^*}}$ are taken as decisions

$$p_{ijm}^{s^*} \approx p_{ijm}^{n+1}, u_{ijm}^{n+1^{s^*}} \approx u_{ijm}^{n+1}, v_{ijm}^{n+1^{s^*}} \approx v_{ijm}^{n+1}, w_{ijm}^{n+1^{s^*}} \approx w_{ijm}^{n+1},$$

difference approximations of expressions

$$\frac{\partial}{\partial x}(\mu\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y}) + \frac{\partial}{\partial z}(\mu\frac{\partial u}{\partial z}),$$

$$k\frac{\partial}{\partial y}[u\left(u^2 + w^2\right)^{\frac{1}{2}}\mu\frac{\partial v}{\partial y}],\ k\frac{\partial}{\partial z}[u\left(u^2 + v^2\right)^{\frac{1}{2}}\mu\frac{\partial w}{\partial z}],$$

$$\frac{\partial}{\partial x}(\mu\frac{\partial v}{\partial x}) + \frac{\partial}{\partial y}(\mu\frac{\partial v}{\partial y}) + \frac{\partial}{\partial z}(\mu\frac{\partial v}{\partial z}),$$

$$k\frac{\partial}{\partial x}[v\left(u^2 + w^2\right)^{\frac{1}{2}}\mu\frac{\partial u}{\partial x}],\ k\frac{\partial}{\partial z}[v\left(u^2 + v^2\right)^{\frac{1}{2}}\mu\frac{\partial w}{\partial z}],$$

$$\frac{\partial}{\partial x}(\mu\frac{\partial w}{\partial x}) + \frac{\partial}{\partial y}(\mu\frac{\partial w}{\partial y}) + \frac{\partial}{\partial z}(\mu\frac{\partial w}{\partial z}),$$

$$k\frac{\partial}{\partial x}[w\left(u^2 + w^2\right)^{\frac{1}{2}}\mu\frac{\partial u}{\partial x}], k\frac{\partial}{\partial y}[w\left(u^2 + w^2\right)^{\frac{1}{2}}\mu\frac{\partial v}{\partial y}],$$

are included accordingly into $F_{uijm}^n, F_{vijm}^n, F_{wijm}^n$, it is advisable to approximate them by samples (ref. /2/):

$$\frac{\partial}{\partial x}\left[\lambda\frac{\partial\Phi}{\partial x}\right]_{ijm}^n = \frac{2}{h_{xi+1} + h_{xi}}[\frac{(\lambda_{i+1\,jm}^n + \lambda_{ijm}^n)(\Phi_{i+1\,jm}^n - \Phi_{ijm}^n)}{2h_{xi+1}} -$$

$$-\frac{(\lambda_{ijm}^n + \lambda_{i-1\,jm}^n)(\Phi_{ijm}^n - \Phi_{i-1\,jm}^n)}{2h_{xi}}] + O(h_{xi+1} - h_{xi}) + O(\hbar_{xi}^2),$$

$$\frac{\partial}{\partial y}\left[\lambda\frac{\partial\Phi}{\partial y}\right]_{ijm}^{n}=\frac{2}{h_{yj+1}+h_{yj}}[\frac{(\lambda_{ij+1m}^{n}+\lambda_{ijm}^{n})(\Phi_{ij+1m}^{n}-\Phi_{ijm}^{n})}{2h_{yj+1}}-$$

$$-\frac{(\lambda_{ijm}^{n}+\lambda_{ij-1m}^{n})(\Phi_{ijm}^{n}-\Phi_{ij-1m}^{n})}{2h_{yj}}]+O(h_{yj+1}-h_{yj})+O(\hbar_{yj}^{2}),$$

$$\frac{\partial}{\partial z}\left[\lambda\frac{\partial\Phi}{\partial z}\right]_{ijm}^{n}=\frac{2}{h_{zm+1}+h_{zm}}[\frac{(\lambda_{ijm+1}^{n}+\lambda_{ijm}^{n})(\Phi_{ijm+1}^{n}-\Phi_{ijm}^{n})}{2h_{zm+1}}-$$

$$-\frac{(\lambda_{ijm}^{n}+\lambda_{ijm-1}^{n})(\Phi_{ijm}^{n}-\Phi_{ijm-1}^{n})}{2h_{zm}}]+O(h_{zm+1}-h_{zm})+O(\hbar_{zm}^{2})$$

These approximations are suitable for discontinuous coefficients /5/.

Note. Generalization of stated iteration algorithms by three-dimensional problems in cylindrical, spherical and other systems of coordinates does not cause special difficulties and is performed similarly to the above-stated (ref. /1/).

Literature

1. Джакупов К.Б. Простые разностные схемы для уравнений гидро-аэротермодинамики. –Алматы: изд-во КазНУ им.Аль-Фараби, 2004г. С.246.
2. Джакупов К.Б.О некоторых разностных схемах для уравнений Навье-Стокса // Чис.мет.мех.спл.ср., ВЦ СО АН СССР, №1,т.2, 1971г. С.17-26.
3. Джакупов К.Б.Численный расчет ламинарного течения вязкой несжимаемой жидкости в кожухе с вращающимися дисками// Известия СО АН СССР, сер.техн.н.вып.1, №3, 1977г., с.18-28.
4. Устименко Б.П., Джакупов К.Б., Кроль В.О. Численное моделирование аэродинамики и теплообмена в топочных и технологических устройствах.- Алма-Ата: изд-во «Наука», 1986г.
5. Самарский А.А. Введение в теорию разностных схем.-М.: Наука, 1970г. С.552.
6. Джакупов К.Б.Численный расчет турбулентного обтекания пластины с применением уравнений для пульсаций// Известия СО АН СССР, сер.техн.н.вып.1, №4, 1985г., с.61-67.

Chapter 10. PARADOX OF COMPUTED "NEGATIVE" GAS DENSITY. EFFECTIVE METHODS OF SOLVING COMPRESSIBLE GAS EQUATIONS

§1. Paradox of negative gas density is caused by incorrect realization of law of mass conservation

In numerical solution of equations of compressible gas by explicit difference schemes of *Brailovskaya, Davydov,* by implicit schemes of /1/, /2/, /4/, /5/, /12/ and others, obtained are *negative* values of gas density ρ, which conflicts physics of this value, measured in SI by relation of kilogram by cubic meter (mass of any matter is positive). Reason for this is incorrect realization of difference analogue in continuity equation

$$\frac{\partial\rho}{\partial t} + div\rho\vec{v} = 0 \tag{1}$$

used in the above-mentioned works for direct calculation of density ρ by difference scheme of the following type

$$\Lambda\frac{\rho^{n+1}-\rho^{n}}{\tau_{n+1}} + (div\rho^{n}\vec{v}^{n})_h = 0, \tag{2}$$

where Λ - linear invertible operator; $(div\rho^{n}\vec{v}^{n})_h -$ difference approximation $div\rho\vec{v}$. Indeed, from (2) calculated is density on the following time layer by inversion of operator Λ:

$$\rho^{n+1} = \rho^{n} - \tau_{n+1}\Lambda^{-1}(div\rho^{n}\vec{v}^{n})_h, \tag{3}$$

It is obvious, that in those points of flow, where $\Lambda^{-1}(div\rho^{n}\vec{v}^{n})_h$ preserves in the counting process a positive sign, density ρ^{n+1} with the growth n will be decreased and will become negative, which is absurd, since it is deprived of sense, or vice versa, increase up to unreal large values, if $\Lambda^{-1}(div\rho^{n}\vec{v}^{n})_h > 0.$

G.I.Marchuk, for eliminating appearing fluctuations increasing when $t\to\infty$ by amplitude, even in observance of conditions of stability, proposed density ρ profile smoothing on each time layer

(first towards τ, then by θ) by formula $\tilde{\rho} = \rho_m + (\rho_{m-1} - 2\rho_m + \rho_{m+1})\delta$, and for stability of counting it is necessary that $0 \le \delta \le 0{,}25$.

These facts and a number of other facts brought up question of fallacy of density calculation by algorithms of type (3) and search of in principle other algorithms of solving equations of viscous compressible gas.

Such algorithm was grounded in /6/,/14/ and this book, where is was concluded that realization of law of mass conservation (1) is performed via pressure function

$$\frac{\partial^2}{\partial t^2}\left(\frac{p}{RT}\right) = \sum_{i=1}^{3}\left\{\frac{\partial^2 p}{\partial x_i^2} + \frac{\partial^2}{\partial x_i^2}\left(\frac{1}{3}\mu - \mu'\right)div\vec{v} + \right.$$

$$\left. + \frac{\partial}{\partial x_i}[div(\rho v_i \vec{v}) - div(\mu grad v_i) - \rho F_i]\right\},$$

i.e. closed system of difference equations for calculation of pressure is constructed from difference analogues of continuity equation. Having in mind that pressure gradient is included into viscous fluid equation, both compressible as well as incompressible equally, only for compressible gas density is defined by pressure and gas temperature $\rho = \frac{P}{RT}$, and for incompressible media $\rho = const$, technology of constructing these finite-difference equations is absolutely similar with the technique stated in this book for incompressible fluid equations.

In monograph /6/, with the use of method energy inequalities. For the first time studied were issues of stability and one-valued solvability of family of semi-implicit and semi-explicit schemes for equations of *Navier-Stokes,* thermal conduction and diffusions with account to chemical reaction of burning in cylindrical coordinates system. Closed system of difference equations was obtained for pressure. From foreign works on numerical solution of compressible gas equations, it is necessary to notice article of *D.G.Lilley* “Simple method of calculation velocities and pressure in strongly swirling flows” // Rocket engineering and astronautics.1976, v.14, June, p.57-67. This work is close in main idea but in it, in three-dimensional

problems used are 4 half-step displaced in relation to each other spatial grids, in two-dimensional – one grid less, while in the methods of the given chapter *one grid* is used, which considerably simplifies theoretical analysis and programming. Half-step displaced grids contain values of unknown functions beyond physical domain of flow, and their definition introduces *uncorrectable* errors in results of calculations, which is discussed in details in **Chapter 8.**

§ 2. Semi-implicit differential scheme of complete equations of compressible viscid gas

Principles of building of semi-implicit, semi-nonimplicit, method of rhythmic steps and variable directions of schemes for equations *Navier-Stokes* of perfect gas were expounded in monograph /6/.

In this paragraph technology /6/ is applied for new system equations, well-founded in **Chapter 1:**

$$\rho(\frac{\partial v_e}{\partial t}+\sum_{m=1}^{3} v_m \frac{\partial v_e}{\partial x_m})+\frac{\partial p}{\partial x_e}=\rho F_e+$$

$$+\sum_{m=1}^{3}\frac{\partial}{\partial x_m}(\mu\frac{\partial v_e}{\partial x_m})-\frac{\partial}{\partial x_e}[(\frac{1}{3}\mu-\mu')div\vec{v}],\ e=1,2,3,$$

$$\rho c_v(\frac{\partial T}{\partial t}+\sum_{m=1}^{3} v_m \frac{\partial T}{\partial x_m})=div(\lambda gradT)+$$

$$+\mu\sum_{e=1}^{3}\sum_{m=1}^{3}(\frac{\partial v_e}{\partial x_m})^2-pdiv\vec{v}-(\frac{1}{3}\mu-\mu')(div\vec{v})^2,$$

$$\frac{\partial \rho}{\partial t}+\sum_{e=1}^{3}\frac{\partial \rho v_e}{\partial x_e}=0,\ p=R\rho T,\mu=\mu(T),\lambda=\lambda(T),$$

under entry conditions

$$T\big|_{t=0}=d_T,\rho\big|_{t=0}=d_\rho,v_e\big|_{t=0}=d_e,e=1,2,3,$$

and boundary conditions on boundary of district of integration

$$T\big|_S=\varphi_T,v_e\big|_S=\varphi_e,e=1,2,3$$

Notice, that for pressure p entry and boundary conditions don't put, for density of gas p don't put boundary conditions, there are not

necessary. First, offer semi-implicit differential schemes are organized thus, that values of pressure are calculated in appropriate

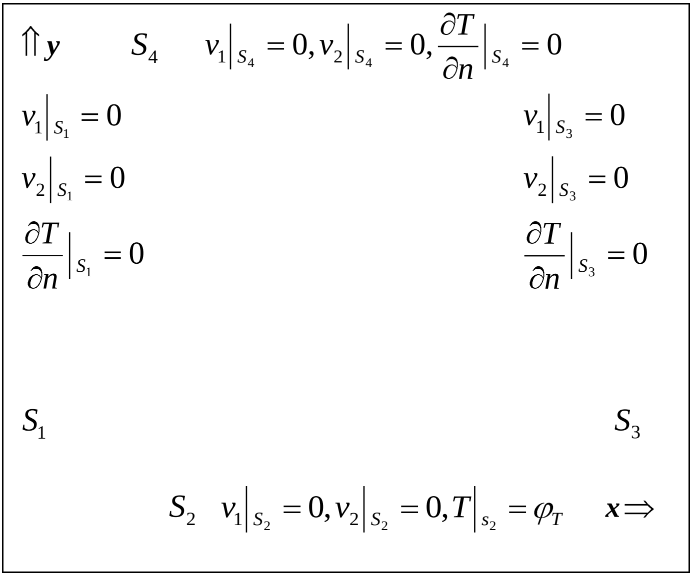


Рис.1

boundary node S_h of net domain by iterative numerical algorithm. Therefore density of gas is calculated in full field, including boundary, from equation status $\rho = p/(RT)$. By way of example on drawing 1, list problem statement about convection by heating of gas below $T\big|_{S_2} = \varphi_T$, rest walls of field are heat-proof. Clearly, that in this task any assignment of boundary conditions for pressure will appreciate absolutely other task.

Secondly, for numerical solution of these equations completely apply technology of building and realization of schemes, stated in previous chapter 8. In technology of chapter 8 for *pressure* proved *difference* boundary conditions from continuity equation. The same circumstance takes place and for equations of compressed gas.

In tasks of flow of bodies unlimited stream possible decide complete equations in net domain $0 \le i \le N_1, 0 \le j \le N_2$, $0 \le k \le N_3$ put, for example, in tasks of calculations of flow by direction x_1 by

way of one from boundary conditions by $i=N_1$ equality of zero of 2 derivative from desired function

$$\partial^2 f / \partial x_1^2 = 0 \qquad (1)$$

On nonuniform mesh (1) approximate by formula

$$[(f_{N_1jk}^{n+1} - f_{N_1-1jk}^{n+1})/h_{1N_1} - (f_{N_1-1jk}^{n+1} - f_{N_1-2jk}^{n+1})/h_{1N_1-1}]\cdot 2/(h_{1N_1} + h_{1N_1-1}) = 0, \quad (2)$$

on analytical grid $h_{1N_1} = h_{1N_1-1} = h_1$ from (2) prove

$$f_{N_1jk}^{n+1} = 2 f_{N_1-1jk}^{n+1} - f_{N_1-2jk}^{n+1}, \qquad (3)$$

Approximate all convection terms, including

$$(v_1^n \partial v_e^n / \partial x_1)_{ijk}, e = 1,2,3, (v_1^n \partial T^n / \partial x_1)_{ijk}$$

possible make by formula without "diagrammatical tenacity"

$$v_{mijk}^n (\frac{\partial \omega}{\partial x_m})_{ijk} = \frac{|v_m^n| + v_m^n}{2} \omega_{\bar{x}_m^0} + \frac{v_m^n - |v_m^n|}{2} \omega_{x_m^0} \equiv v_m^n \omega_{\ddot{x}_m}, m = 1,2,3,$$

or use principles from monotonous schemes of chapter 6 in some cases by more simple scheme

$$v_{mijk}^n (\frac{\partial \omega}{\partial x_m})_{ijk} = \frac{|v_m^n| + v_m^n}{2} \omega_{\bar{x}_m} + \frac{v_m^n - |v_m^n|}{2} \omega_{x_m}, m = 1,2,3,$$

but in any case with central difference

$$v_{mijk}^n (\frac{\partial \omega}{\partial x_m})_{ijk} = v_m^n \frac{\omega_{\bar{x}_m} + \omega_{x_m}}{2}, m = 1,2,3,$$

they let to incorrect results (*to serrated decision*).

In semi-implicit scheme gradients of pressure and component of speed in continuity equation approximate with *central* differences and take on upper layer *n+1* outward iteration, but all rest terms of equation on lower layer of time t_n :

$$\rho^n[(v_e^{n+1} - v_e^n)/\tau_{n+1} + \sum_{m=1}^{3} v_m^n v_{e\ddot{x}_m}^n] + p_{\tilde{x}_e}^{n+1} = \rho^n F_e + \qquad (4)$$

$$+ \sum_{m=1}^{3} (\mu^n v_{ex_m}^n)_{\dot{x}_m} - [\sum_{m=1}^{3} (\mu^n / 3 - \mu'^n) v_{mx_m}^n)_{\bar{x}_e}], e = 1,2,3,$$

$$\rho^n c_v[(T^{n+1}-T^n)/\tau_{j+1}+\sum_{m=2}^{3}v_m^n T_{\ddot{x}_m}^n]=\sum_{m=1}^{3}(\lambda^n T_{x_m}^n)_{\dot{x}_m}+$$

$$+\mu^n\sum_{e=1}^{3}\sum_{m=1}^{3}(v_{e\tilde{x}_m}^n)^2-p^n\sum_{e=1}^{3}v_{e\tilde{x}_e}^n-(\frac{1}{3}\mu^n-\mu'^n)(\sum_{e=1}^{3}v_{e\tilde{x}_e}^n)^2 \qquad (5)$$

Schemes (4), (5) noticed for indexes $1\le i\le N_1-1, 1\le j\le N_2-1$, $1\le k\le N_3-1$. **In particular, possible use monotonous schemes.** In an effort to simplification of statement applied approximation "without diagrammatical tenacity".

1⁰. Scheme of continuity equation

In schemes (4) gradients of pressure are replaced by *central difference derivations*. By stated in /6/ principle coordinated approximation, derivations in continuity equation also approximate by central *difference derivations*:

$$(\rho_{ijk}^{n+1}-\rho_{ijk}^n)/\tau_{n+1}+(\rho^n v_1^{n+1})_{\tilde{x}_1}+(\rho^n v_2^{n+1})_{\tilde{x}_2}+(\rho^n v_3^{n+1})_{\tilde{x}_3}=0, \qquad (6)$$

$$i=1,..N_1,\ j=1,...,N_2-1, k=1,...,N_3-1$$

In boundary nodes assigned temperature and speed components

$$T^{n+1}\Big|_{S_h}=\varphi_T^{n+1}, v_e^{n+1}\Big|_{S_h}=\varphi_e^{n+1}, m=1,2,3$$

In boundary nodes $i=0,\forall(j,k); j=0,\forall(i,k); k=0,\forall(i,j)$ continuity equation approximate by technology /1/ difference forward

$$\rho_t^n+(\rho^n v_e^{n+1})_{x_e}+\sum_{\substack{m=1\\ m\ne l}}^{3}\frac{\partial\rho^n v_m^{n+1}}{\partial x_m}=0, x\in S_{eh}, e=1,2,3, \qquad (7)$$

in boundary node $i=N_1,\forall(j,k); j=N_2,\forall(i,k); k=N_3,\forall(i,j)$ $k=N_3,\forall(i,j)$ approximate differences rearward:

$$\rho_t^n+(\rho^n v_e^{n+1})_{\bar{x}_e}+\sum_{\substack{m=1\\ m\ne l}}^{3}\frac{\partial\rho^n v_m^{n+1}}{\partial x_m}=0, x\in \bar{S}_{eh}, e=1,2,3, \qquad (7э)$$

For example, in boundary nodes $j=0,\forall(i,k)$ (7) transform to type

$$(\rho_{i0k}^{n+1}-\rho_{i0k}^{n})/\tau_{n+1}+(\rho_{i+10k}^{n}\varphi_{1i+10k}^{n+1}-\rho_{i-10k}^{n}\varphi_{1i-10k}^{n+1})/(h_{1i+1}+h_{1i})+(\rho_{i1k}^{n}v_{2i1k}^{n+1}- \quad (8)$$

$$-\rho_{i0k}^{n}\varphi_{2i0k}^{n+1})/h_{21}+(\rho_{i0k+1}^{n}\varphi_{3i0k+1}^{n+1}-\rho_{i0k-1}^{n}\varphi_{3i0k-1}^{n+1})/(h_{3k+1}+h_{3k})=0,\ j=0,\forall(i,k)$$

In boundary node $k=0,\forall(i,j)$ notice (7) take type

$$\begin{aligned}&(\rho_{ij0}^{n+1}-\rho_{ij0}^{n})/\tau_{n+1}+(\rho_{i+1j0}^{n}\varphi_{1i+1j0}^{n+1}-\rho_{i-1j0}^{n}\varphi_{1i-1j0})/(h_{1i+1}+h_{1i})+(\rho_{ij+10}^{n}\varphi_{2ij+10}^{n+1}-\\&-\rho_{ij-10}^{n}\varphi_{2ij-10}^{n+1})/(h_{2j+1}+h_{2j})+(\rho_{ij1}^{n}v_{3ij1}^{n+1}-\rho_{ij0}^{n}\varphi_{3ij0}^{n+1})/h_{31}=0,k=0,\forall(i,j)\end{aligned} \quad (9)$$

In boundary nodes $i=N_1,\forall(j,k)$ notice (7э) take concrete form

$$\begin{aligned}&(\rho_{N_1jk}^{n+1}-\rho_{N_1jk}^{n})/\tau_{n+1}+(\rho_{N_1jk}^{n}\varphi_{1N_1jk}^{n+1}-\rho_{N_1-1jk}^{n}v_{1N_1-1jk}^{n+1})/h_{1N_1}+\\&+(\rho_{N_1j+1k}^{n}\varphi_{2N_1j+1k}^{n+1}-\rho_{N_1j-1k}^{n}\varphi_{2N_1j-1k}^{n+1})/(h_{2j+1}+h_{2j})+\\&+(\rho_{N_1k+1}^{n}\varphi_{3N_1jk+1}^{n+1}-\rho_{N_1k-1}^{n}\varphi_{3N_1jk-1}^{n+1})/(h_{3k+1}+h_{3k})=0,\ j=0,\forall(i,k)\end{aligned}$$

And so forth on other areas of border.

2⁰. Realization of state equation $p=\rho RT$

By this clear scheme (5) count field of temperature T^{n+1}, therefore state equation possible present in type $\rho^n=p^n/(RT^n)$, in difference continuity equation (6), (7), (7э), (8), (9) and other put $\rho_{ijk}^{n+1}=p_{ijk}^{n+1}/(RT_{ijk}^{n+1})$, because ρ^n well-known value of density with previous layout of time t_n. It necessary take into account, that state equation $p=R\rho T$ take place in all points of net domain $0\le i\le N_1, 0\le j\le N_2, 0\le k\le N_3$.

3⁰. Difference equation for pressure

Previously equation (4) increase on step by time τ_{n+1}, after that present in type

$$\rho_{ijk}^{n}v_{eijk}^{n+1}=[-p_{\tilde{x}_e}^{n+1}-Q_{eijk}^{n}]\tau_{n+1},\ \ e=1,2,3,$$

$$Q_{eijk}^{n}=\rho_{ijk}^{n}[-\tau_{n+1}^{-1}\,v_{eijk}^{n}+\sum_{m=1}^{3}v_{mijk}^{n}v_{e\ddot{x}_m}^{n}-F_e\ \ ]-\sum_{m=1}^{3}(\mu_m^n v_{ex_m}^n)_{\dot{x}_m}+$$

$$+\sum_{m=1}^{3}(\mu^n/3-\mu'^n)v^n_{mx_m})_{\bar{x}_e}, \qquad (10)$$

$$1\le i\le N_1-1, 1\le j\le N_2-1, 1\le k\le N_3-1$$

With purpose of using of results previous chapter come back to initial indications

$$v_1\equiv u, v_2\equiv v, v_3\equiv w, \varphi_1\equiv\varphi_u,$$

$$\varphi_2\equiv\varphi_v, \varphi_3\equiv\varphi_w, h_1\equiv h_x, h_2\equiv h_y, h_3\equiv h_z, N_1\equiv N_x, N_2\equiv N_y, N_3\equiv N_z,$$

$$F^n_{uijk}=-\tau_{n+1}Q^n_{1ijk}, F^n_{vijk}=-\tau_{n+1}Q^n_{2ijk}, F^n_{wijk}=-\tau_{n+1}Q^n_{3ijk}$$

In this indications (10) take type

$$\rho^n_{ijk}u^{n+1}_{ijk}=F^n_{uijk}-\tau_{n+1}\frac{p^{n+1}_{i+1jk}-p^{n+1}_{i-1jk}}{h_{xi+1}+h_{xi}},$$

$$\rho^n_{ijk}v^{n+1}_{ijk}=F^n_{vijk}-\tau_{n+1}\frac{p^{n+1}_{ij+1k}-p^{n+1}_{ij-1k}}{h_{yj+1}+h_{yj}}, \qquad (11)$$

$$\rho^n_{ijk}w^{n+1}_{ijk}=F^n_{wijk}-\tau_{n+1}\frac{p^{n+1}_{ijk+1}-p^{n+1}_{ijk-1}}{h_{zk+1}+h_{zk}}$$

Equation for pressure p^{n+1}_{ijk} prove by way of substitution indifference analogues of continuity equation (6), (7), (7э), (8), (9) and other:

$$[p^{n+1}_{ijk}/(RT^{n+1}_{ijk})-\rho^n_{ijk}]/\tau_{n+1}+$$

$$+\left\{(F^n_{ui+1jk}-\tau_{n+1}\frac{p^{n+1}_{i+2jk}-p^{n+1}_{ijk}}{h_{xi+2}+h_{xi+1}})\frac{1-sign(i-N_x+1.5)}{2}+\right.$$

$$+\frac{1+sign(i-N_x+1.5)}{2}\varphi^{n+1}_{uN_xjk}-\frac{1+sign(i-1.5)}{2}(F^n_{ui-1jk}-\tau_{n+1}\frac{p^{n+1}_{ijk}-p^{n+1}_{i-2jk}}{h_{xi}+h_{xi-1}})-$$

$$\left.-\frac{1-sign(i-1.5)}{2}\varphi^{n+1}_{u0jk}\right\}\frac{1}{h_{xi+1}+h_{xi}}+\left\{\frac{1+sign(j-N_y+1.5)}{2}\varphi^{n+1}_{viN_yk}+\right.$$

$$+(F^{n}_{vij+1k}-\tau_{n+1}\frac{p^{n+1}_{ij+2k}-p^{n+1}_{ijk}}{h_{yj+2}+h_{yj+1}})\frac{1-sign(j-N_y+1.5)}{2}-$$

$$-(F^{n}_{vij-1k}-\tau_{n+1}\frac{p^{n+1}_{ijk}-p^{n+1}_{ij-2k}}{h_{yj}+h_{yj-1}})\frac{1+sign(j-1.5)}{2}-$$

$$-\frac{1-sign(j-1.5)}{2}\varphi^{n+1}_{vi0k}\ \Big\}\frac{1}{h_{yj+1}+h_{yj}}+ \qquad (12)$$

$$+\ \Big\{\frac{1+sign(k-N_z+1.5)}{2}\varphi^{n+1}_{wijN_z}+$$

$$+(F^{n}_{wijk+1}-\tau_{n+1}\frac{p^{n+1}_{ijk+2}-p^{n+1}_{ijk}}{h_{zk+2}+h_{yj+1}})\frac{1-sign(k-N_z+1.5)}{2}-$$

$$-(F^{n}_{wijk-1}-\tau_{n+1}\frac{p^{n+1}_{ijk}-p^{n+1}_{ijk-2}}{h_{zk}+h_{zk-1}})\frac{1+sign(k-1.5)}{2}-$$

$$-\frac{1-sign(k-1.5)}{2}\varphi^{n+1}_{wij0}\ \Big\}\frac{1}{h_{zk+1}+h_{zk}}=0,$$

$$i=1,\dots,N_x-1,\ j=1,\dots,N_y-1,\ j=1,\dots,N_y-1,$$

boundary conditions for pressure prove in type:

$$[p^{n+1}_{N_x jk}/(RT^{n+1}_{N_x jk})-\rho^{n}_{N_x jk}]/\tau_{n+1}+$$

$$+\frac{1}{h_{xN_x}}\{\varphi^{n+1}_{uN_x jk}-(F^{n}_{uN_x-1jk}-\tau_{n+1}\frac{p^{n+1}_{N_x jk}-p^{n+1}_{N_x-2jk}}{h_{xN_x}+h_{xN_x-1}})\}+$$

$$+(\frac{\partial\rho^{n}\varphi^{n+1}_{v}}{\partial y})^{n+1}_{N_x jk}+(\frac{\partial\rho^{n}\varphi^{n+1}_{w}}{\partial z})^{n+1}_{N_x jk}=0,\ j=1,\dots,N_y-1, k=1,\dots,N_z-1;$$

$$[p^{n+1}_{0jk}/(RT^{n+1}_{0jk})-\rho^{n}_{0jk}]/\tau_{n+1}+\frac{1}{h_{x1}}\{(F^{n}_{u1jk}-$$

$$-\tau_{n+1}\frac{p_{2jk}^{n+1}-p_{0jk}^{n+1}}{h_{x2}+h_{x1}})-\varphi_{u0jk}^{n+1}\}+(\frac{\partial\rho^{n}\varphi_{v}^{n+1}}{\partial y})_{0jk}^{n+1}+(\frac{\partial\rho^{n}\varphi_{w}^{n+1}}{\partial z})_{0jk}^{n+1}=0,$$

$$j=1,\ldots,N_y-1,k=1,\ldots,N_z-1;$$

$$[p_{iN_yk}^{n+1}/(RT_{iN_yk}^{n+1})-\rho_{iN_yk}^{n}]/\tau_{n+1}+\frac{1}{h_{yN_y}}\{\varphi_{viN_yk}^{n+1}-(F_{viN_y-1k}^{n}-$$

$$-\tau_{n+1}\frac{p_{iN_yk}^{n+1}-p_{iN_y-2k}^{n+1}}{h_{yN_y}+h_{yN_y-1}})\}+(\frac{\partial\rho^{n}\varphi_{u}^{n+1}}{\partial x})_{iN_yk}^{n+1}+(\frac{\partial\rho^{n}\varphi_{w}^{n+1}}{\partial z})_{iN_yk}^{n+1}=0,$$

$$i=1,\ldots,N_x-1,k=1,\ldots,N_z-1;\ ,$$

$$[p_{i0k}^{n+1}/(RT_{i0k}^{n+1})-\rho_{i0k}^{n}]/\tau_{n+1}+\frac{1}{h_{y1}}\{(F_{vi1k}^{n}-\tau_{n+1}\frac{p_{i2k}^{n+1}-p_{i0k}^{n+1}}{h_{y2}+h_{y1}})-$$

$$-\varphi_{vi0k}^{n+1}\}+(\frac{\partial\rho^{n}\varphi_{ui0k}^{n+1}}{\partial x})+(\frac{\partial\rho^{n}\varphi_{wi0k}^{n+1}}{\partial z})=0,i=1,\ldots,N_x-1,k=1,\ldots,N_z-1;$$

$$[p_{ijN_z}^{n+1}/(RT_{ijN_z}^{n+1})-\rho_{ijN_z}^{n}]/\tau_{n+1}+\frac{1}{h_{zN_z}}\{\varphi_{wijN_z}^{n+1}-(F_{wijN_z-1}^{n}-$$

$$-\tau_{n+1}\frac{p_{ijN_z}^{n+1}-p_{ijN_z-2}^{n+1}}{h_{zN_z}+h_{zN_z-1}})\}+(\frac{\partial\rho^{n}\varphi_{u}^{n+1}}{\partial x})_{ijN_z}^{n+1}+(\frac{\partial\rho^{n}\varphi_{v}^{n+1}}{\partial y})_{ijN_z}^{n+1}=0,$$

$$i=1,\ldots,N_x-1,\ j=1,\ldots,N_y-1;\ ,$$

$$[p_{ij0}^{n+1}/(RT_{ij0}^{n+1})-\rho_{ij0}^{n}]/\tau_{n+1}+\frac{1}{h_{z1}}\{\ (F_{wij1}^{n}-\ \tau_{n+1}\frac{p_{ij2}^{n+1}-p_{ij0}^{n+1}}{h_{z2}+h_{z1}})-$$

$$-\varphi_{vij0}^{n+1}\}+(\frac{\partial\rho^{n}\varphi_{uij0}^{n+1}}{\partial x})+(\frac{\partial\rho^{n}\varphi_{wij0}^{n+1}}{\partial z})=0,i=1,\ldots,N_x-1,\ j=1,\ldots,N_y-1;$$

§3. Global iterative method of calculation of *pressure*

In this method in iterative process is included equations (10) on every iterative layout by *s:*

$$\rho^n_{ijk} v^{s,n+1}_{eijk} = [-p^{s,n+1}_{\tilde{x}_e} - Q^n_{eijk}]\tau_{n+1}, \quad v^{s,n+1}_{eijk}\Big|_S = \varphi^{n+1}_{eijk}, \qquad (13)$$

$$e = 1,2,3, 1 \le i \le N_1 - 1, 1 \le j \le N_2 - 1, 1 \le k \le N_3 - 1,$$

which in complex type assist in difference continuity equation:

$$(1+\theta A^n_{ijk})(p^{s+1,n+1}_{ijk} - p^{s,n+1}_{ijk})/\theta + [p^{s,n+1}_{ijk}/(RT^{n+1}_{ijk}) - \rho^n_{ijk}]/\tau_{n+1} +$$

$$+(\rho^n v_1^{s,n+1})_{\tilde{x}_1} + (\rho^n v_2^{s,n+1})_{\tilde{x}_2} + (\rho^n v_3^{s,n+1})_{\tilde{x}_3} = 0, \qquad (14)$$

where easily calculate from (12)

$$A^n_{ijk} = \frac{1}{\tau_{n+1} RT^{n+1}_{ijk}} + [\frac{1}{h_{xi+2}+h_{xi+1}} \frac{1-sign(i-N_x+1.5)}{2} +$$

$$+\frac{1}{h_{xi}+h_{xi-1}} \frac{1+sign(i-1.5)}{2}] \frac{\tau_{n+1}}{h_{xi+1}+h_{xi}} +$$

$$+[\frac{1}{h_{yj+2}+h_{yj+1}} \frac{1-sign(j-N_y+1.5)}{2} + \frac{1}{h_{yj}+h_{yj-1}} \frac{1+sign(j-1.5)}{2}] \frac{\tau_{n+1}}{h_{yj+1}+h_{yj}} +$$

$$+[\frac{1}{h_{zk+2}+h_{zk+1}} \frac{1-sign(k-N_z+1.5)}{2} + \frac{1}{h_{zk}+h_{zk-1}} \frac{1+sign(k-1.5)}{2}] \frac{\tau_{n+1}}{h_{zk+1}+h_{zk}},$$

$$1 \le i \le N_1 - 1, 1 \le j \le N_2 - 1, 1 \le k \le N_3 - 1$$

Iterations in boundary nodes $j = 0$**:**

$$(1+\theta A^n_{i0k})(p^{s+1,n+1}_{i0k} - p^{s,n+1}_{i0k})/\theta + [p^{s+1,n+1}_{i0k}/(RT^{n+1}_{i0k}) - \rho^n_{i0k}]/\tau_{n+1} +$$

$$+(\rho^n_{i+10k}\varphi^{n+1}_{1i+10k} - \rho^n_{i-10k}\varphi^{n+1}_{1i-10k})/(h_{1i+1}+h_{1i}) + (\rho^n_{i1k} v^{s,n+1}_{2i1k} - \qquad (15)$$

$$-\rho^n_{i0k}\varphi^{n+1}_{2i0k})/h_{21} + (\rho^n_{i0k+1}\varphi^{n+1}_{3i0k+1} - \rho^n_{i0k-1}\varphi^{n+1}_{3i0k-1})/(h_{3k+1}+h_{3k}) = 0,$$

$$A^n_{i0k} = \frac{1}{\tau_{n+1} RT^{n+1}_{i0k}} + \frac{\tau_{n+1}}{h_{21}} \frac{1}{h_{22}+h_{21}}, i = 1,\ldots,N_1 - 1, k = 1,\ldots,N_3 - 1$$

Iterations in boundary nodes $k = 0$**:**

$$(1+\theta A^n_{ij0})(p^{s+1,n+1}_{ij0} - p^{s,n+1}_{ij0})/\theta + [p^{s+1,n+1}_{ij0}/(RT^{n+1}_{ij0}) - \rho^n_{ij0}]/\tau_{n+1} +$$

$$+(\rho^n_{i+1j0}\varphi^{n+1}_{1i+1j0} - \rho^n_{i-1j0}\varphi^{n+1}_{1i-1j0})/(h_{1i+1}+h_{1i}) + (\rho^n_{ij+10}\varphi^{n+1}_{2ij+10} - \qquad (16)$$

$$-\rho^n_{ij-10}\varphi^{n+1}_{2ij-10})/(h_{2j+1}+h_{2j}) + (\rho^n_{ij1} v^{s,n+1}_{3ij1} - \rho^n_{ij0}\varphi^{n+1}_{3ij0})/h_{31} = 0,$$

$$A_{ij0}^{n}=\frac{1}{\tau_{n+1}RT_{ij0}^{n+1}}+\frac{\tau_{n+1}}{h_{31}}\frac{1}{h_{32}+h_{31}},\ i=1,\dots,N_1-1,\ j=1,\dots,N_2-1$$

Iterations in boundary nodes $i=0$ **:**

$$(1+\theta A_{0jk}^{n})(p_{0jk}^{s+1,n+1}-p_{0jk}^{s,n+1})/\theta+[p_{0jk}^{s+1,n+1}/(RT_{0jk}^{n+1})-\rho_{0jk}^{n}]/\tau_{n+1}+$$
$$+(\rho_{1jk}^{n}v_{11jk}^{s,n+1}-\rho_{0jk}^{n}\varphi_{10jk}^{n+1})/h_{11}+(\rho_{0j+1k}^{n}\varphi_{20j+1k}^{n+1}- \qquad (17)$$
$$-\rho_{0j-1k}^{n}\varphi_{20j-1k}^{n+1})/(h_{2j+1}+h_{2j})+(\rho_{0jk+1}^{n}\varphi_{30jk+1}^{n+1}-\rho_{0jk-1}^{n}\varphi_{30jk-1}^{n+1})/(h_{3k+1}+h_{3k})=0,$$
$$A_{0jk}^{n}=\frac{1}{\tau_{n+1}RT_{0jk}^{n+1}}+\frac{\tau_{n+1}}{h_{11}}\frac{1}{h_{12}+h_{11}},\ j=1,\dots,N_2-1,k=1,\dots,N_3-1$$

Iterations in boundary nodes $i=N_1$ **:**

$$(1+\theta A_{N_1jk}^{n})(p_{N_1jk}^{s+1,n+1}-p_{N_1jk}^{s,n+1})/\theta+[p_{N_1jk}^{s+1,n+1}/(RT_{N_1jk}^{n+1})-\rho_{N_1jk}^{n}]/\tau_{n+1}+$$
$$+(\rho_{N_1jk}^{\mathrm{n}}\varphi_{1N_1jk}^{\mathrm{n+1}}-\rho_{N_1-1jk}^{\mathrm{n}}v_{1N_1-1jk}^{\mathrm{s,n+1}})/h_{1N_1}+(\rho_{N_1j+1k}^{\mathrm{n}}\varphi_{2N_1j+1k}^{n+1}- \qquad (18)$$
$$-\rho_{N_1j-1k}^{n}\varphi_{2N_1j-1k}^{n+1})/(h_{2j+1}+h_{2j})+(\rho_{N_1jk+1}^{n}\varphi_{3N_1jk+1}^{n+1}-\rho_{N_1jk-1}^{n}\varphi_{3N_1jk-1}^{n+1})/(h_{3k+1}+h_{3k})=0,$$
$$A_{N_1jk}^{n}=\frac{1}{\tau_{n+1}RT_{N_1jk}^{n+1}}+\frac{\tau_{n+1}}{h_{1N_1}}\frac{1}{h_{1N_1}+h_{1N_1-1}},\ j=1,\dots,N_2-1,k=1,\dots,N_3-1$$

Iterations in boundary nodes $j=N_2$ **:**

$$(1+\theta A_{iN_2k}^{n})(p_{iN_2k}^{s+1,n+1}-p_{iN_2k}^{s,n+1})/\theta+[p_{iN_2k}^{s+1,n+1}/(RT_{iN_2k}^{n+1})-\rho_{iN_2k}^{n}]/\tau_{n+1}+$$
$$+(\rho_{i+1N_2k}^{n}\varphi_{1i+1N_2k}^{n+1}-\rho_{i-1N_2k}^{n}\varphi_{1i-1N_2k}^{n+1})/(h_{1i+1}+h_{1i})+(\rho_{iN_2k}^{n}\varphi_{2iN_2k}^{n+1}- \qquad (19)$$
$$-\rho_{iN_2-1k}^{n}v_{2iN_2-1k}^{s,n+1})/h_{2N_2}+(\rho_{iN_2k+1}^{n}\varphi_{3iN_2k+1}^{n+1}-\rho_{iN_2k-1}^{n}\varphi_{3iN_2k-1}^{n+1})/(h_{3k+1}+h_{3k})=0,$$
$$A_{iN_2k}^{n}=\frac{1}{\tau_{n+1}RT_{iN_2k}^{n+1}}+\frac{\tau_{n+1}}{h_{2N_2}}\frac{1}{h_{2N_2}+h_{2N_2-1}},\ 1\le k\le N_3-1, 1\le i\le N_1-1$$

Iterations in boundary nodes $k=N_3$ **:**

$$(1+\theta A_{ijN_3}^{n})(p_{ijN_3}^{s+1,n+1}-p_{ijN_3}^{s,n+1})/\theta+[p_{ijN_3}^{s+1,n+1}/(RT_{ijN_3}^{n+1})-\rho_{ijN_3}^{n}]/\tau_{n+1}+$$
$$+(\rho_{i+1jN_3}^{n}\varphi_{1i+1jN_3}^{n+1}-\rho_{i-1jN_3}\varphi_{1i-1jN_3}^{n+1})/(h_{1i+1}+h_{1i})+(\rho_{ij+1N_3}^{n}\varphi_{2ij+1N_3}^{n+1}- \qquad (20)$$
$$-\rho_{ij-1N_3}^{n}\varphi_{2ij-1N_3}^{n+1})/(h_{2j+1}+h_{2j})+(\rho_{ijN_3}^{n}\varphi_{3ijN_3}^{n+1}-\rho_{ijN_3-1}^{n}v_{3ijN_3-1}^{s,n+1})/h_{3N_3}=0,$$

$$A^n_{ijN_3} = \frac{1}{\tau_{n+1} RT^{n+1}_{ijN_3}} + \frac{\tau_{n+1}}{h_{3N_3}} \frac{1}{h_{3N_3} + h_{3N_3-1}}, \ 1 \le i \le N_1 - 1, 1 \le j \le N_2 - 1$$

Parameter of iteration select from interval $0 < \theta < 0{,}5$. In all without execution applied above iteration processes, exactness calculation of field of pressure limited by exactness of execution of continuity equation.

§4. Indexed family semi-implicit schemes

Set above scheme is special case of family semi-implicit schemes, well-founded in /6/.

$$\rho^n_{ijk}[(v^{n+1}_{eijk} - v^n_{eijk})/\tau_{n+1} + \sum_{m=1}^{3} v^n_{mijk} v^n_{e\ddot{x}_m}] + (\overline{\alpha}_e p^{n+1}_{\bar{x}_e} + \overline{\beta}_e p^{n+1}_{x_e})/2 = \rho^n_{ijk} F_e +$$

$$+ \sum_{m=1}^{3} (\mu^n_m v^n_{ex_m})_{\dot{x}_m} - [\sum_{m=1}^{3} \{(\frac{\mu^n}{3} - \mu'^n) v^n_{mx_m}\}_{\bar{x}_e}], e = 1,2,3,$$

$$\rho^n_{ijk} c_v [(T^{n+1}_{ijk} - T^n_{ijk})/\tau_{n+1} + \sum_{m=1}^{3} v^n_{mijk} T^n_{\ddot{x}_m}] = \sum_{m=1}^{3} (\lambda^n_m T^n_{x_m})_{\dot{x}_m} +$$

$$+ \mu^n_{ijk} \sum_{e=1}^{3} \{\sum_{m=1}^{3} [(v^n_{e\bar{x}_m} + v^n_{ex_m})/2]^2\} -$$

$$- p^n_{ijk} \sum_{e=1}^{3} (v^n_{e\bar{x}_e} + v^n_{ex_e})/2 - (\frac{1}{3}\mu^n_{ijk} - \mu'^n_{ijk})\{\sum_{e=1}^{3} [(v^n_{e\bar{x}_e} + v^n_{ex_e})/2]\}^2 \ ,$$

$$1 \le i \le N_1 - 1, 1 \le j \le N_2 - 1, 1 \le k \le N_3 - 1$$

with parameters $\overline{\alpha}_e$ and $\overline{\beta}_e = 1 - \overline{\alpha}_e, \quad e = 1,2,3$, by technology of global iterations

$$\rho^n_{ijk} v^{s,n+1}_{eijk} = [-(\overline{\alpha}_e p^{s,n+1}_{\bar{x}_e} + \overline{\beta}_e p^{s,n+1}_{x_e}) - Q^n_{eijk}]\tau_{n+1}, v^{s,n+1}_{eijk}\Big|_{\sigma} = \varphi^{n+1}_{eijk}, \quad (21)$$

$$e = 1,2,3, 1 \le i \le N_1 - 1, 1 \le j \le N_2 - 1, 1 \le k \le N_3 - 1,$$

which assist in difference continuity equations in inward nodes

$$(1+\theta\overline{A}_{ijk}^{n})(p_{ijk}^{s+1,n+1}-p_{ijk}^{s,n+1})/\theta+[p_{ijk}^{s,n+1}/(RT_{ijk}^{n+1})-\rho_{ijk}^{n}]/\tau_{n+1}+$$
$$+[\overline{\alpha}_1(\rho^n v_1^{s,n+1})_{x_1}+\overline{\beta}_1(\rho^n v_1^{s,n+1})_{\bar{x}_1}]+[\overline{\alpha}_2(\rho^n v_2^{s,n+1})_{x_2}+\overline{\beta}_2(\rho^n v_2^{s,n+1})_{\bar{x}_2}]+$$
$$+[\overline{\alpha}_3(\rho^n v_3^{s,n+1})_{x_3}+\overline{\beta}_3(\rho^n v_3^{s,n+1})_{\bar{x}_3}]=0, \qquad (22)$$
$$1\le i\le N_1-1, 1\le j\le N_2-1, 1\le k\le N_3-1,$$

Iterations in boundary nodes $j=0$**:**

$$\overline{\alpha}_2\{(1+\theta\overline{A}_{i0k}^{n})(p_{i0k}^{s+1,n+1}-p_{i0k}^{s,n+1})/\theta+[p_{i0k}^{s+1,n+1}/(RT_{i0k}^{n+1})-\rho_{i0k}^{n}]/\tau_{n+1}+$$
$$+(\rho_{i+10k}^{n}\varphi_{1i+10k}^{n+1}-\rho_{i0k}^{n}\varphi_{1i0k}^{n+1})/h_{1i+1}+(\rho_{i1k}^{n}v_{2i1k}^{s,n+1}-$$
$$-\rho_{i0k}^{n}\varphi_{2i0k}^{n+1})/h_{21}+(\rho_{i0k+1}^{n}\varphi_{3i0k+1}^{n+1}-\rho_{i0k}^{n}\varphi_{3i0k}^{n+1})/h_{3k+1}\}=0, \qquad (23)$$
$$i=1,\dots,N_1-1, k=1,\dots,N_3-1, j=0,$$

Iterations in boundary nodes $k=0$**:**

$$\overline{\alpha}_3\{(1+\theta\overline{A}_{ij0}^{n})(p_{ij0}^{s+1,n+1}-p_{ij0}^{s,n+1})/\theta+[p_{ij0}^{s+1,n+1}/(RT_{ij0}^{n+1})-\rho_{ij0}^{n}]/\tau_{n+1}+$$
$$+(\rho_{i+1j0}^{n}\varphi_{1i+1j0}^{n+1}-\rho_{ij0}^{n}\varphi_{1ij0}^{n+1})/h_{1i+1}+(\rho_{ij+10}^{n}\varphi_{2ij+10}^{n+1}-$$
$$-\rho_{ij0}^{n}\varphi_{2ij0}^{n+1})/h_{2j+1}+(\rho_{ij1}^{n}v_{3ij1}^{s,n+1}-\rho_{ij0}^{n}\varphi_{3ij0}^{n+1})/h_{31}\}=0, \qquad (24)$$
$$i=1,\dots,N_1-1, j=1,\dots,N_2-1, k=0$$

Iterations in boundary nodes $i=0$**:**

$$\overline{\alpha}_1\{(1+\theta\overline{A}_{0jk}^{n})(p_{0jk}^{s+1,n+1}-p_{0jk}^{s,n+1})/\theta+[p_{0jk}^{s+1,n+1}/(RT_{0jk}^{n+1})-\rho_{0jk}^{n}]/\tau_{n+1}+$$
$$+(\rho_{1jk}^{n}v_{11jk}^{s,n+1}-\rho_{0jk}^{n}\varphi_{10jk}^{n+1})/h_{11}+(\rho_{0j+1k}^{n}\varphi_{20j+1k}^{n+1}- \qquad (25)$$
$$-\rho_{0jk}^{n}\varphi_{20jk}^{n+1})/h_{2j+1}+(\rho_{0jk+1}^{n}\varphi_{30jk+1}^{n+1}-\rho_{0jk}^{n}\varphi_{30jk}^{n+1})/h_{3k+1}\}=0,$$
$$i=0, j=1,\dots,N_2-1, k=1,\dots,N_3-1$$

Iterations in boundary nodes $i=N_1$**:**

$$\bar{\beta}_1\{(1+\theta\bar{A}^n_{N_1jk})(p^{s+1,n+1}_{N_1jk}-p^{s,n+1}_{N_1jk})/\theta+[p^{s+1,n+1}_{N_1jk}/(RT^{n+1}_{N_1jk})-\rho^n_{N_1jk}]/\tau_{n+1}+$$

$$+(\rho^{n}_{N_1jk}\varphi^{n+1}_{1N_1jk}-\rho^{n}_{N_1-1jk}\,v^{s,n+1}_{1N_1-1jk})/h_{1N_1}+(\rho^{n}_{N_1jk}\varphi^{n+1}_{2N_1jk}- \quad (26)$$

$$-\rho^n_{N_1j-1k}\varphi^{n+1}_{2N_1j-1k})/h_{2j}+(\rho^n_{N_1jk}\varphi^{n+1}_{3N_1jk}-\rho^n_{N_1jk-1}\varphi^{n+1}_{3N_1jk-1})/h_{3k}\}=0,$$

$$i=N_1,\ j=1,\ldots,N_2-1,k=1,\ldots,N_3-1$$

Iterations in boundary nodes $j = N_2$ **:**

$$\bar{\beta}_2\{(1+\theta\bar{A}^n_{iN_2k})(p^{s+1,n+1}_{iN_2k}-p^{s,n+1}_{iN_2k})/\theta+[p^{s+1,n+1}_{iN_2k}/(RT^{n+1}_{iN_2k})-\rho^n_{iN_2k}]/\tau_{n+1}+$$

$$+(\rho^n_{iN_2k}\varphi^{n+1}_{1iN_2k}-\rho^n_{i-1N_2k}\varphi^{n+1}_{1i-1N_2k})/h_{1i}+(\rho^n_{iN_2k}\varphi^{n+1}_{2iN_2k}-$$

$$-\rho^n_{iN_2-1k}v^{s,n+1}_{2iN_2-1k})/h_{2N_2}+(\rho^n_{iN_2k}\varphi^{n+1}_{3iN_2k}-\rho^n_{iN_2k-1}\varphi^{n+1}_{3iN_2k-1})/h_{3k}\}=0,$$

$$j=N_2,1\le k\le N_3-1,1\le i\le N_1-1 \qquad (27)$$

Iterations in boundary nodes $k = N_3$ **:**

$$\bar{\beta}_3\{(1+\theta\bar{A}^n_{ijN_3})(p^{s+1,n+1}_{ijN_3}-p^{s,n+1}_{ijN_3})/\theta+[p^{s+1,n+1}_{ijN_3}/(RT^{n+1}_{ijN_3})-\rho^n_{ijN_3}]/\tau_{n+1}+$$

$$+(\rho^n_{ijN_3}\varphi^{n+1}_{1ijN_3}-\rho_{i-1jN_3}\varphi^{n+1}_{1i-1jN_3})/h_{1i}+(\rho^n_{ijN_3}\varphi^{n+1}_{2ijN_3}- \quad (28)$$

$$-\rho^n_{ij-1N_3}\varphi^{n+1}_{2ij-1N_3})/h_{2j}+(\rho^n_{ijN_3}\varphi^{n+1}_{3ijN_3}-\rho^n_{ijN_3-1}v^{s,n+1}_{3ijN_3-1})/h_{3N_3}\}=0,$$

$$1\le i\le N_1-1,k=N_3,1\le j\le N_2-1$$

By values $\bar{\alpha}_e = 0.5, \bar{\beta}_e = 0.5, e = 1,2,3$ from (21) – (28) prove scheme with global iterations (13) – (20). By other values of parameters $\bar{\alpha}_e$ and $\bar{\beta}_e = 1-\bar{\alpha}_e, e = 1,2,3$ take place analogous schemes, which differ each other *interconsisted approximations of gradients of pressure and continuity equation*.

In all without include applied above iteration processes, exactness of calculation of field of pressuse limited by exactness of execution of continuity equation.

Notice. Equality of zero some parameter $\overline{\alpha}_e$, $\overline{\beta}_e$, $e = 1,2,3$ means, that in this nodes continuity equation isn't applied for execution of pressure.

§5. Technology building of scheme for march equations of compressible perfect gas

March method of calculation of flow is applied in those cases, when in equations 2 derivatives by one from spatial coordinates possible disregard. For example, believe, that axle "x_1" direct by flow, put equal zero $\partial^2 v_e / \partial x_1^2 = 0, e = 1,2,3, \partial^2 T / \partial x_1^2 = 0$ or disregard diffusion

flows $\frac{\partial}{\partial x_1}(\mu \frac{\partial v_e}{\partial x_1}) = 0,\ \frac{\partial}{\partial x_1}(\lambda \frac{\partial T}{\partial x_1}) = 0.$ In orthogonal to flow section x_{1i-1} consider well-known (from boundary conditions or already calculated by algorithm) values desired functions $v_{ei-1jk}, T_{i-1jk}, p_{i-1jk}$ and other, need find them values in section x_1, that is $v_{eijk}, T_{ijk}, p_{ijk}$ and other. For calculation stationary flows difference schemes /6/ apply in type iteration algorithm, when decision $v_{eijk}, T_{ijk}, p_{ijk}$ and other, prove as limits

$$\lim_{n\to\infty} v_{eijk}^n = v_{eijk}, \lim_{n\to\infty} T_{ijk}^n = T_{ijk}, \lim_{n\to\infty} p_{ijk}^n = p_{ijk},$$

by execution conditions $\lim\limits_{n\to\infty} \frac{v_{eijk}^{n+1} - v_{eijk}^n}{\tau_{n+1}} = 0,\ \lim\limits_{n\to\infty} \frac{T_{ijk}^{n+1} - T_{ijk}^n}{\tau_{n+1}} = 0,$

and the like, it is enough to put in indicated schemes flows equal zero

$$\frac{\partial}{\partial x_1}(\mu \frac{\partial v_e}{\partial x_1}) = 0, e = 1,2,3, \frac{\partial}{\partial x_1}(\lambda \frac{\partial T}{\partial x_1}) = 0,$$

and convection terms by direction x_1 approximate difference backwards

$$(v_1^n \frac{\partial v_e^n}{\partial x_1})_{ijk} = v_{1ijk}^n (v_{eijk}^n - v_{ei-1jk}^n)/h_{1i} \equiv v_1^n v_{\bar{x}_1}^n,$$

that is put in schemes /6/ $\alpha_1^n = 0{,}5 + 0{,}5 sign(v_{1ijk}^n) = 1,$ as $v_{1ijk}^n > 0,$ like manner $(v_1^n \partial T^n / \partial x_1)_{ijk} = v_1^n \cdot T_{\bar{x}_1}^n$ and so on.

Constituent gradient of pressure $\partial p / \partial x_1$ in section x_1, approximate difference derivative backwards $p_{\bar{x}_1}^{j+1}$, that is in schemes /6/necessary put $\alpha_1 = 1, \beta_1 = 0$, for two other constituent $\partial p / \partial x_m$, $m = 2{,}3$ apply principle interconsisted approximation with continuity equation. In continuity equation $(\frac{\partial(\rho v_1)}{\partial x_1})_{ijk}$ approximate in section x_1, in view direction of flow in positive direction of axle x_1 also difference backwards:

$$(\frac{\partial(\rho v_1)}{\partial x_1})_{ijk} \cong (\rho v_1)_{\bar{x}_1}^n = (\rho_{ijk}^n v_{1ijk}^n - \rho_{i-1jk} v_{1i-1jk}) h_{1i}^{-1},$$

where $\rho_{i-1jk} v_{1i-1jk}$ calculated or well-known magnitudes, therefore in them don't put upper index n. In lineage march methods upper index n don't put: $v_{ei-1jk}^n = v_{ei-1jk}$ and so on.

On every iteration layout t_{n+1} necessary execution inward iterations for executions p_{ijk}^{n+1} as limit iteration

$$\lim_{s \to \infty} p_{ijk}^{s,n+1} = p_{ijk}^{n+1}$$

additional upper index s conform to number of s – iteration. For beginning field of iteration $s = 0$ is selected value p_{ijk}^n :

$$p_{ijk}^{s,n+1} = p_{ijk}^{n}, 0 \le i \le N_1, 0 \le j \le N_2, 0 \le k \le N_3 .$$

§6. Method of determinants in explicit iterationless scheme of solving stationary march equations

Applied is stationary system of march equations

$$\rho \sum_{m=1}^{3} v_m \frac{\partial v_1}{\partial x_m}) + RT\frac{\partial \rho}{\partial x_1} + R\rho \frac{\partial T}{\partial x_1} = \rho F_1 +$$

$$+ \sum_{m=2}^{3} \frac{\partial}{\partial x_m}(\mu \frac{\partial v_1}{\partial x_m}) - \frac{\partial}{\partial x_1}[(\frac{1}{3}\mu - \mu')\sum_{m=2}^{3}\frac{\partial v_m}{\partial x_m})], \; e = 1,$$

$$\rho \sum_{m=1}^{3} v_m \frac{\partial v_e}{\partial x_m} + \frac{\partial p}{\partial x_e} = \rho F_e + \sum_{m=2}^{3} \frac{\partial}{\partial x_m}(\mu \frac{\partial v_e}{\partial x_m}) -$$

$$- \frac{\partial}{\partial x_e}[(\frac{1}{3}\mu - \mu')\sum_{m=2}^{3}\frac{\partial v_m}{\partial x_m})], \; e = 2,3,$$

$$\rho c_v \sum_{m=1}^{3} v_m \frac{\partial T}{\partial x_m}) = \sum_{m=2}^{3} \frac{\partial}{\partial x_m}(\lambda \frac{\partial T}{\partial x_m}) + \mu \sum_{e=1}^{3}\sum_{m=1}^{3}(\frac{\partial v_e}{\partial x_m})^2 -$$

$$- p div\vec{v} - (\frac{1}{3}\mu - \mu')(div\vec{v})^2,$$

$$\rho \frac{\partial v_1}{\partial x_1} + v_1 \frac{\partial \rho}{\partial x_1} + \sum_{e=2}^{3} \frac{\partial \rho v_e}{\partial x_e} = 0, \; p = R\rho T, \mu = \mu(T), \lambda = \lambda(T)$$

where substitution is preliminarily made

$$\partial p / \partial x_1 = RT \partial \rho / \partial x_1 + R\rho \partial T / \partial x_1 ,$$

Shortly denoted are backward difference derivatives

$$U_T = T_{\bar{x}_1} \equiv (T_{ijk} - T_{i-1jk})/h_{1i}, U_1 = v_{1\bar{x}_1} \equiv (v_{1ijk} - v_{1i-1jk})/h_{1i},$$

$$U_\rho = \rho_{\bar{x}_1} \equiv (\rho_{ijk} - \rho_{i-1jk})/h_{1i},$$

approximating $\partial T / \partial x_1$, $\partial v_1 / \partial x_1$, $\partial \rho / \partial x_1$.

For calculations U_T, U_1, U_ρ the following linear system comprising three equations was made, composed of scheme for v_1

$$\rho_{i-1jk}[(v_{1i-1jk}U_1+\sum_{m=2}^{3}v_{mi-1jk}(v_{1\breve{x}_m})_{i-1jk}]+RT_{i-1jk}U_\rho+R\rho_{i-1jk}U_T=$$

$$=\rho_{i-1jk}F_{1i-1jk}+\sum_{m=2}^{3}[(\mu_m v_{1\bar{x}_m})_{\dot{x}_m}]_{i-1jk}-\sum_{m=2}^{3}[(\mu/3-\mu')v_{mx_m})_{\bar{x}_e}]_{i-1jk},$$

from schemes for continuity equation

$$v_{1i-1jk}U_\rho+\rho_{i-1jk}U_1+\sum_{e=2}^{3}[(\rho v_{e\bar{x}_e})_{i-1jk}+(\rho v_{ex_e})_{i-1jk}]/2=0,$$

and energy balance

$$\rho_{i-1jk}[v_{1i-1jk}U_T+\sum_{m=2}^{3}v_{mi-1jk}(T_{\breve{x}_m})_{i-1jk}]=\sum_{m=2}^{3}[(\lambda_m T_{\bar{x}_m})_{\dot{x}_m}]_{i-1jk}+$$

$$+\mu_{i-1jk}\sum_{e=1}^{3}\{[(v_{ei-1jk}-v_{ei-2jk})/h_{1ii}]^2+\sum_{m=2}^{3}[(v_{e\bar{x}_m}+v_{ex_m})_{i-1jk}/2]^2\}-$$

$$-p_{i-1jk}\ [U_1+\sum_{e=2}^{3}[(v_{e\bar{x}_e}+v_{ex_e})_{i-1jk}/2]\}-$$

$$-(\frac{1}{3}\mu-\mu')\ \{(v_{1i-1jk}-v_{1i-1jk})/h_{1i}+\sum_{e=2}^{3}[(v_{e\bar{x}_e}+v_{ex_e})_{i-1jk}/2]\}^2,$$

$$i\ge 1, 1\le j\le N_2-1, 1\le k\le N_3-1$$

In the initial cross-section it is assumed

$$v_{e-1jk}=v_{e0jk}=\varphi_{e0jk}, e=1,2,3.$$

Obtained was linear system of type

$$a_{11}U_1+a_{12}U_\rho+a_{13}U_T=b_1,$$

$$a_{21}U_1+a_{22}U_\rho+a_{23}U_T=b_2$$

$$a_{31}U_1+a_{32}U_\rho+a_{33}U_T=b_3$$

Is solved by method of determinants, then found are unknown quantities in section x_{1i} for all j,k: v_{1ijk}, ρ_{ijk}, T_{ijk}: $v_{1ijk}=v_{1i-1jk}+h_{1i_1}U_1, \rho_{ijk}=\rho_{i-1jk}+h_{1i}U_\rho, T_{ijk}=T_{ii-1jk}+h_{1i}U_T$, after that,

by explicit schemes calculated are $p_{ijk} = \rho_{ijk} RT_{ijk}$ and velocity components $v_{eijk}, e = 2{,}3$:

$$\rho_{ijk}[(v_{1ijk}(v_{eijk} - v_{ei-1jk})/h_{1i} + \sum_{m=2}^{3} v_{mi-1jk}(v_{e\breve{x}_m})_{i-1jk}] +$$

$$+ (p_{\bar{x}_e} + p_{x_e})_{ijk}/2 = \rho_{ijk} F_{1ijk} +$$

$$+ \sum_{m=2}^{3}[(\mu_m v_{e\bar{x}_m})_{\dot{x}_m}]_{i-1jk} - \sum_{m=1}^{3}[(\mu/3 - \mu')v_{mx_m})_{\bar{x}_e}]_{i-1jk}, e = 2{,}3$$

In initial conditions, at input, it is necessary to set density $v_1 = v_{input}, v_2 = v_{input}, v_3 = v_{input}, T = T_{input}, \rho = \rho_{input}$. Under flow periodicity, for instance, by $x_m, m = 2$ or *m=3 index* i_m changed within limits $1 \le i_m \le N_m$. In boundary nodes $i_m = 0, i_m = N_m, m = 2{,}3$ calculation of density engages: either continuity equation, or condition of symmetry, on solid impermeable surfaces extrapolation formulae or other physical conditions. If in energy balance equation $pdiv\vec{v}$ =0, there arises a system comprising only 2 equations

$$a_{11}U_1 + a_{12}U_\rho = b_1,$$
$$a_{21}U_1 + a_{22}U_\rho = b_2$$

Main disadvantage of march methods of such type is that for calculation of pressure are not applied difference analogues of **§3 of** canonical equation

$$\frac{\partial^2}{\partial t^2}(\frac{p}{RT}) = \sum_{i=1}^{3}\{\frac{\partial^2 p}{\partial x_i^2} + \frac{\partial^2}{\partial x_i^2}[(\frac{1}{3}\mu - \mu')div\vec{v}] +$$

$$+ \frac{\partial}{\partial x_i}[div(\rho v_i \vec{v}) - div(\mu grad v_i) - \rho F_i]\}$$

Literature

1.Chorin A.J. Numerical solution of the Navier-Stokes equations// Math. Comp., 23, 1968, p.341-354.

2.Полежаев В.И. Численное решение системы двумерных нестационарных уравнений Навье-Стокса для сжимаемого газа// Изв.СО АН СССР, сер.техн.н., 1967г., №13, вып.3, с.73-80.
3.Джакупов К.Б., Гончаров А.И. Численное моделирование диффузионного горения в закрученных потоках двухфакельной топки // Сибирский физико-технический журнал. 1991г., вып.6/91, с.83-93.
4.Махвиладзе Г.М., Щербак С.Б.Численный метод исследования нестационарных пространственных движений сжимаемого газа//ИФЖ, 1980г., т.38, №3, с.528-537.
5.Борисов А.В. Ковеня В.М. Применение неявной разностной схемы для расчета внутренних течений вязкого газа// Числ.мет. мех.спл.ср., 1976г., т.7, №4, с.36-47.
6.Джакупов К.Б. Простые разностные схемы для уравнений гидроаэро-термодинамики. - Алматы, изд-во КазНУ им. Аль-Фараби,2004г., с.246.
7.Андерсон, Дейл, Плетчер. Вычислительная гидромеханика и теплообмен.Т.1, 2.- М.: "Мир", 1982г. С.723.
8.Марчук Г.И. Численные методы в прогнозе погоды.-Л., изд-во Гидромет,1967г., с.356.
9.Джакупов К.Б. О новых уравнениях динамики вязкой жидкости // Тезисы докладов междунар. науч. конф. "Пробл. теор. и прикл. механики", Алматы, Казахстан, 1-2 марта, 2006г..
10.Джакупов К.Б. Циклическая пятиточечная прогонка // Известия АН КазССР. Сер.физ.-мат.н..1977г., №1, с.19-24.
11.Джакупов К.Б. Лабораторный практикум по вычислительной гидродинамике. – Алматы: Изд-во КазНУ им.Аль-Фараби. 2006г. С.224.
12.Данаев Н.Т. Докторская диссертация. – Алматы, КазНУ им.Аль-Фараби, 1998г.
13.Джакупов К.Б. Докторская диссертация. – Новосибирск, Институт теплофизики им.С.С.Кутателадзе СОАН СССР, 1990г.
14.Джакупов К.Б. Численный расчет течений газа при малых числах Маха во входной секции и камере охлаждения топочного модуля // Известия СОАН СССР, сер.техн.н..1985г., вып.2, №10, с.27-33.

15.Джакупов К.Б. О некоторых разностных схемах для уравнений Навье-Стокса // Чис.мет.мех.спл.ср., ВЦ СО АН СССР, №1, т.2, 1971г. С.17-26.

Chapter 11. UNIVERSAL METHOD OF ACCELERATION OF CONVERGENCE OF ITERATION ALGORITHMS

Let for find decision *p* equation

$$Ap = f, \tag{1}$$

where *f* – given quantity, *A* – nondegenerate linear operator, for which is inverse A^{-1}, apply some *coveregence* iteration algorithm:

$$B(p^{s+1} - p^s)/r_s = Ap^s - f,\ s = 0,1,\ldots,s^* \tag{2}$$

As a rule, with the exception of p^s more part of proved information about decision p with previous steps of iteration $p^\nu, \nu = 0,1,\ldots,s-1$ rests without further applying in mind that don't use return.

Basic idea offer universal method concludes exactly in return using of results p^{s-i+1} of previous iterations (see *Jakupov K.B.* /1/)

$$p^{s+2} = \sum_{i=0}^{n} \gamma_i p^{s-i+1}, \gamma_i = c\text{onst},\ \sum_{i=0}^{n} \gamma_i = 1,\ n \le s+1, \tag{3}$$

as contains with some kind of degree of exactness of valuable information about desired solution p equation (1).

In (3) coefficients $\gamma_i = c\text{onst}$ are found from condition of minimization square of gilbert's form of residual

$$\left\| R^{s+2} \right\|^2 = \left\| Ap^{s+2} - f \right\|_D^2,$$

some self-conjugate positive certain operator-generated D, for the sake of simplicity suppose in further *D=E, E* – identical (unit) operator.

Definition. Acceleration, made by lineage combination (3), named *m-cyclical acceleration, where m – quantity of different from zero of parameters* γ_i, which are subject to calculate with indicated method.

This definition supposes opportunity of preliminary selective equalization to zero some parameters $\gamma_{\bar{i}}, \bar{i} \neq 0$.

Let p^s - s-iteration p; $R^s = Ap^s - f$ - residual s-iteration; f, p, p^s – essence component of gilbert's space H; (f,φ) – scalar product in H; form of component $\|p\| = \sqrt{(p,p)}$;

H is field of definition and field of values of operator A.

Theorem 1 (basic). *In m=n+1 – cyclical acceleration*

$$p^{s+2} = \sum_{i=0}^{n} \gamma_i p^{s-i+1}, \gamma_i = \text{const}, \ \sum_{i=0}^{n} \gamma_i = 1, \ 1 \le n \le s+1, \tag{4}$$

parameters $\gamma_i, i = 1,\ldots,n$ *calculated as decision of system n lineage equations*

$$\gamma_i \left\|R^{s-i+1} - R^{s+1}\right\|^2 + \sum_{\substack{j=1 \\ j \neq i}}^{n} \gamma_i (R^{s-i+1} - R^{s+1}, R^{s-j+1} - R^{s+1}) = \\ = (R^{s+1}, R^{s-i+1} - R^{s+1}), \ \ \mathrm{i} = 1,\ldots,\mathrm{n}, \tag{5}$$

are necessary conditions of existence of extremum of function

$$\left\|R^{s+2}\right\|^2 = \left\|Ap^{s+2} - f\right\|^2 = \left\|A\sum_{i=0}^{n} \gamma_i p^{s-i+1} - f\right\|,$$

by this take place inequality $\| R^{s+2} \| \le \| R^{s+1} \|$, that is *residuals don't increase.*

P r o v i n g. Take action by operator A on two parts (4) and subtract f:

$$Ap^{s+2} - f = A\sum_{i=0}^{n} \gamma_i p^{s-i+1} - f = \sum_{i=0}^{n} \gamma_i Ap^{s-i+1} - f\sum_{i=0}^{n} \gamma_i = \\ = \sum_{i=0}^{n} \gamma_i (Ap^{s-i+1} - f), Ap^{s+2} - f = \sum_{i=0}^{n} \gamma_i (Ap^{s-i+1} - f)$$

Enter in last expression of residual, find

$$R^{s+2}=\gamma_0 R^{s+1}+\sum_{i=1}^{n}\gamma_i R^{s-i+1}=(1-\sum_{i=1}^{n}\gamma_i)R^{s+1}+\sum_{i=1}^{n}\gamma_i R^{s-i+1}=$$
$$=R^{s+1}+\sum_{i=1}^{n}\gamma_i(R^{s-i+1}-R^{s+1})$$

Further, by definition of form, have

$$\left\|R^{s+2}\right\|^2=\left\|R^{s+1}\right\|^2+\sum_{i=1}^{n}\gamma_i^2\left\|R^{s-i+1}-R^{s+1}\right\|^2+2\sum_{i=1}^{n}\gamma_i\,(R^{s+1},R^{s-i+1}-R^{s+1})+$$
$$+2\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}\gamma_i\gamma_j(R^{s-i+1}-R^{s+1},R^{s-j+1}-R^{s+1}) \qquad (6)$$

From expression (6) follow necessary conditions of existence of extremum $\partial\left\|R^{s+2}\right\|^2/\partial\gamma_i=0, i=1,2,\ldots,n$ in type of system of lineage equations (5). Apparently, enough conditions of minimum $\partial^2\left\|R^{s+2}\right\|^2/\partial\gamma_i^2\geq 0, i=1,2,\ldots,n$ also are executed. Execution of inequality $\|R^{s+2}\|\leq\|R^{s+1}\|$ is proved by method of mathematical induction below. Show more importance consequences from basic theorem.

Consequense 1. *In di-cyclic* (*m=2,i=1) acceleration*

$$p^{s+2}=\gamma_0 p^{s+1}+\gamma_1 p^s, \gamma_0+\gamma_1=1 \qquad (7)$$

parameters γ_0,γ_1 are executed by formulas

$$\gamma_1=\frac{(R^{k+1},R^{k+1}-R^k)}{\left\|R^{k+1}-R^k\right\|^2},\gamma_0=1-\gamma_1=-\frac{(R^k,R^{k+1}-R^k)}{\left\|R^{k+1}-R^k\right\|^2} \qquad (8)$$

For conclusion of formulas (8) enough to put in (5) (5) *n=1,i=1*.

Acceleration (7) equivalent of method of minimal residuals (m.m.r.) *Krasnoselcky-Krein*. In this method

$$p^{s+1}=p^s+r_s B^{-1}R^s, R^{s+1}=R^s+r_s AB^{-1}R^s, \qquad (9)$$

parameter r_s is found from condition of minimum $\left\|R^{s+1}\right\|^2$ and equals

$$r_s = -\frac{(R^s, AB^{-1}R^s)}{\left\|AB^{-1}R^s\right\|^2} \tag{9'}$$

With other side, in view (7) and (9)

$$p^{s+2} = \gamma_0 p^{s+1} + \gamma_1 p^s = \gamma_0(p^s + \bar{r}_s B^{-1}R^s) + \gamma_1 p^s = p^s + \gamma_0 \bar{r}_s B^{-1}R^s, \tag{10}$$

where $\bar{r}_s \neq r_s$ and in view (8)

$$\gamma_0 \bar{r}_s = -\frac{(R^s, (R^{s+1} - R^s)\bar{r}_s)}{\left\|R^{s+1} - R^s\right\|^2} = -\frac{\left(R^s, \dfrac{R^{s+1} - R^s}{\bar{r}_s}\right)}{\left\|\dfrac{R^{s+1} - R^s}{\bar{r}_s}\right\|^2} = -\frac{(R^s, AB^{-1}R^s)}{\left\|AB^{-1}R^s\right\|^2}$$

Indicate $\gamma_0 \bar{r}_s = \tilde{r}_s$, find, that in (9) and (10) residuals are minimazed equally. This fact is evidence of using formula (7) in methods of minimal residuals give higher rate, which agree with speed one's method. For acceleration of convergence of methods of minimal residuals, multi cyclic equations and dicyclic equations of following types are efficiency.

Theorem 2. *In di-cyclic accelerations (m=2)*

$$p^{s+2} = \gamma_0 p^{s+1} + \gamma_i p^{s-i+1}, \gamma_0 + \gamma_i = 1, i = 1,2,\ldots,s+1 \tag{11}$$

parameters γ_0, γ_i *are calculated by formulas*

$$\gamma_i = \frac{(R^{s+1}, R^{s+1} - R^{s-i+1})}{\| R^{s+1} - R^{s-i+1} \|^2}, \gamma_0 = 1 - \gamma_i, \tag{12}$$

by this take place inequality $\| R^{s+2} \| \leq \| R^{s+1} \|$.

P r o v i n g. In view (11)

$$R^{s+2} = R^{s+1} + \gamma_i (R^{s-i+1} - R^{s+1}),$$

therefore, according with (12) find

$$\left\|R^{s+2}\right\|^2 = \left\|R^{s+1} + \gamma_i (R^{s-i+1} - R^{s+1})\right\|^2 =$$

$$= \left\|R^{s+1}\right\|^2 (1 - \frac{(R^{s+1}, R^{s-i+1} - R^{s+1})^2}{\left\|R^{s+1} - R^{s-i+1}\right\|^2 \left\|R^{s+1}\right\|^2}), \qquad (13)$$

By inequality of *Koshi-Bynyakovskyi*

$$(R^{s+1}, R^{s-i+1} - R^{s+1})^2 \le \left\|R^{s+1}\right\|^2 \left\|R^{s-i+1} - R^{s+1}\right\|^2,$$

in view of (13) follow inequality $|| R^{s+2} || \le || R^{s+1} ||$.

Theorem 3. *In three-cyclic accelerations (m=3)*

$$p^{s+2} = \gamma_0 p^{s+1} + \gamma_i p^{s-i+1} + \gamma_j p^{s-j+1}, i \ne j \ne 0, \gamma_0 + \gamma_i + \gamma_j = 1, \quad (14)$$

$i, j = 1,2,...,s+1$parameters $\gamma_0, \gamma_i, \gamma_j$ are executed by formulas

$$\gamma_i = \frac{(R^{s+1}, R^{s+1} - R^{s+1}_{s-i+1})\left\|R^{s+1}_{s-j+1}\right\|^2 - (R^{s+1}, R^{s+1}_{s-j+1})(R^{s+1}_{s-i+1}, R^{s+1}_{s-j+1})}{\Delta},$$

$$\gamma_j = \frac{(R^{s+1}, R^{s+1} - R^{s+1}_{s-j+1})\left\|R^{s+1}_{s-i+1}\right\|^2 - (R^{s+1}, R^{s+1}_{s-i+1})(R^{s+1}_{s-i+1}, R^{s+1}_{s-j+1})}{\Delta},$$

$$\gamma_0 = 1 - \gamma_i - \gamma_j, \Delta = \left\|R^{s+1}_{s-i+1}\right\|^2 \left\|R^{s+1}_{s-j+1}\right\|^2 - (R^{s+1}_{s-i+1}, R^{s+1}_{s-j+1})^2 \ge 0,$$

$$R^{s+1}_{s-k+1} = R^{s+1} - R^{s-k+1}, k = i, j, \qquad (15)$$

by this $\left\|R^{s+2}\right\| \le \left\|R^{s+1}\right\|$.

P r o v i n g. In view (14)

$$R^{s+2} = R^{s+1} - \gamma_i R^{s+1}_{s-i+1} - \gamma_j R^{s+1}_{s-j+1},$$

$$\left\|R^{s+2}\right\|^2 = \left\|R^{s+1}\right\|^2 + \gamma_i^2 \left\|R^{s+1}_{s-i+1}\right\|^2 + \gamma_j^2 \left\|R^{s+1}_{s-j+1}\right\|^2 - 2\gamma_i (R^{s+1}, R^{s+1}_{s-i+1}) -$$

$$- 2\gamma_j (R^{s+1}, R^{s+1}_{s-j+1}) + 2\gamma_i \gamma_j (R^{s+1}_{s-i+1}, R^{s+1}_{s-j+1}) \qquad (16)$$

Put (15) in (16) and make appropriate transformation, arrive to equality

$$\left\|R^{s+2}\right\|^2=\left\|R^{s+1}\right\|^2(1-\frac{A^2B-2ADC+C^2E}{\Delta\left\|R^{s+1}\right\|^2}), \qquad (17)$$

where

$$A=(R^{s+1},R^{s+1}_{s-i+1}), B=\left\|R^{s+1}_{s-j+1}\right\|^2, E=\left\|R^{s+1}_{s-i+1}\right\|^2$$

$$C=(R^{s+1},R^{s+1}_{s-j+1}),\ \ D=(R^{s+1}_{s-i+1},R^{s+1}_{s-j+1})$$

For that in (17) executed inequality

$$\| R^{s+2} \|\leq\| R^{s+1} \|, \qquad (18)$$

enough prove, that

$$A^2B-2ADC+C^2E\geq 0 \qquad (19)$$

so $A^2B\geq 0, C^2E\geq 0,$ that possible two cases: 1) $ADC\leq 0,$

then inequality (19) executed by apparent way;

2) $ADC>0,$ in this case execution (19) isn't apparently. Proving that by $ADC>0$ take place (19) rests upon that by inequality *Koshi-Bynyakovskyi* $\left|D\right|\leq\sqrt{B}*\sqrt{E}$, therefore,

$$A^2B-2ADC+C^2E\geq A^2B-2sign(AC)\sqrt{B}\cdot\sqrt{E}+C^2E=$$
$$=(A\sqrt{B}\pm C\sqrt{E})^2\geq 0 \text{, etc.}$$

Theorem 4. *In four-cyclic accelerations (m=4)*

$$p^{s+2}=\gamma_0 p^{s+1}+\gamma_i p^{s-i+1}+\gamma_j p^{s-j+1}+\gamma_t p^{s-t+1}, i\neq j\neq t\neq 0,$$
$$\gamma_0+\gamma_i+\gamma_j+\gamma_t=1, i,j,t=1,2,\ldots,s+1 \qquad (20)$$

parameters $\gamma_0,\gamma_i,\gamma_j,\gamma_t$ *are calculated by formulas*

$$\gamma_i = \frac{\begin{vmatrix} Q & B & C \\ P & E & F \\ T & F & M \end{vmatrix}}{\Delta}, \gamma_j = \frac{\begin{vmatrix} A & Q & C \\ B & P & F \\ C & T & M \end{vmatrix}}{\Delta}, \gamma_t = \frac{\begin{vmatrix} A & B & Q \\ B & E & P \\ C & F & T \end{vmatrix}}{\Delta}, \tag{21}$$

$$\gamma_0 = 1 - \gamma_i - \gamma_j - \gamma_t, R^{s+1}_{s-k+1} = R^{s+1} - R^{s-k+1}, k = i, j, t,$$

$$A = \left\| R^{s+1}_{s-i+1} \right\|^2, B = (R^{s+1}_{s-i+1}, R^{s+1}_{s-j+1}), C = (R^{s+1}_{s-i+1}, R^{s+1}_{s-t+1}),$$

$$E = \left\| R^{s+1}_{s-j+1} \right\|^2, F = (R^{s+1}_{s-j+1}, R^{s+1}_{s-t+1}), M = \left\| R^{s+1}_{s-t+1} \right\|^2,$$

$$Q = (R^{s+1}, R^{s+1}_{s-i+1}), P = (R^{s+1}, R^{s+1}_{s-j+1}), T = (R^{s+1}, R^{s+1}_{s-t+1}),$$

$$\Delta = \begin{vmatrix} A & B & C \\ B & E & F \\ C & F & M \end{vmatrix},$$

by this $\| R^{s+2} \| \leq \| R^{s+1} \|$.

Formulas (21) follow from (5). Use method of mathematical induction possible put proving, that in **theorem 4** and in basic theorem take place inequality $\| R^{s+2} \| \leq \| R^{s+1} \|$, as for *n=1* and *n=2* this proved in theorems 2 and 3.

Really, formula (20) can be written in type, which resembles three-cyclic (14):

$$p^{s+2} = \gamma_0 p^{s+1} + \gamma_i p^{s-i+1} + \bar{\gamma} p^{\bar{s}}, \tag{22}$$

$$\bar{\gamma} p^{\bar{s}} = \frac{\gamma_j}{\gamma_j + \gamma_t} p^{s-j+1} + \frac{\gamma_t}{\gamma_j + \gamma_t} p^{s-t+1}, \bar{\gamma} = \gamma_j + \gamma_t$$

If $\gamma_j = -\gamma_t$, that (20) possible write in type

$$p^{s+2} = \gamma_0 p^{s+1} + \gamma_i p^{s-i+1} + \gamma_j p^{\bar{s}}, \; p^{\bar{s}} = p^{s-j+1} - p^{s-t+1} \qquad (23)$$

Formulas (22), (23) formally agree with (14), and that parameters $\gamma_0, \gamma_i, \bar{\gamma}, \gamma_j$ are calculated from conditions of extremum.

Theorem 3 is right for three-cyclic acceleration, therefore $\| R^{s+2} \| \leq \| R^{s+1} \|$, etc. Similar arguments method of mathematical induction lets find, that in basic theorem will take place necessary inequality $\| R^{s+2} \| \leq \| R^{s+1} \|$. Notable property of acceleration (3) is that inequality $\| R^{s+2} \| \leq \| R^{s+1} \|$ executed for any nonvacuous lineage operator A, from which not require so properties, as diagonal prevalence, self-adjointness, positive definiteness and etc., need only convergence of iterations.

Numerical experiments, taken with two-cyclic acceleration by $i \geq 5$ showed, that in some iteration methods acceleration makes them convergence from 1,5 till 9 times faster. In the capacity of illustration below showed tables of exchanges of maximum-form of residual on example of decision of *Dirichlet'*s task for *Puasson* equation on network $0 \leq n \leq N_1, \quad 0 \leq m \leq N_2, N_1 = 20, N_2 = 20$ with two-cyclic acceleration of method of *Krasnoselskyi-Krein* (m-K-K).

In theorem 2 used only two previous iterations p^{s+1} and p^{s-i+1}, for example, for *i=3* $p^{s+2} = \gamma_0 p^{s+1} + \gamma_3 p^{s-2}$ for *i=5* $p^{s+2} = \gamma_0 p^{s+1} + \gamma_5 p^{s-4}$, etc.

For example, applying of two-cyclic acceleration for i=5 made by algorithm:

$$p^1, p^2, p^3, p^4, p^5, \bar{p}^6, p^7, p^8, p^9, p^{10}, p^{11}, p^{12}, \bar{p}^{13}, \ldots, p^{s-4}, p^{s-3}, p^{s-2}, p^{s-1}, p^s, p^{s+1}, \bar{p}^{s+2}, \ldots,$$

Overlined iterations $\bar{p}^6, \bar{p}^{13}, \bar{p}^{20}, \ldots, \bar{p}^{s+2}$ are calculated by formula (11), but non-overlined

$$p^1, p^2, p^4, p^5, p^7 p^8, p^9, p^{10} p^{11}, p^{12}, \ldots,, p^{s-4}, p^{s-3}, p^{s-2}, p^{s-1}, p^s, p^{s+1}, \ldots,$$

calculated by basic (that is speed up) algorithm.

Experiments showed, that advantage by time takes place for two-cyclic accelerations with number $i \geq 5$.

Necessary take into account, that residuals in boundary nodes by edge conditions of *Dirichlet*, wittingly equal zero, by edge conditions of type *fon Neiman* calculated by given algorithm. Effect of acceleration is reached by calculations with two exactness.

Tables of decrease of residual forms $\| R^s \| = \max | R^s_{nm} |$

by two-cyclic acceleration $p^{s+2} = \gamma_0 p^{s+1} + \gamma_i p^{s-i+1}$

with exactness of iteration $\| R^{s^*} \| \leq \varepsilon, \varepsilon = 0{,}01$

Iteration *s*=…	(м-К-К) without acceleration $\| R^s \| = \ldots$, $s^* = 194$	Acceleration *i=5* $\| R^s \| = \ldots$, $s^* = 86$	Acceleration *i=7* $\| R^s \| = \ldots$, $s^* = 97$	Acceleration *i=8* $\| R^s \| = \ldots$, $s^* = 104$
s=0	806,820	806,820	806,820	806,820
s=10	70,398	73,002	74,087	74,903
s=20	55,096	41,048	45,618	44,292
s=30	34,500	14,071	20,214	11,872
s=40	20,966	3,299	6,903	4,039
s=50	12,696	1,067	1,836	1,934
s=60	7,685	0,274	0,828	0,542
s=70	4,651	0,047	0,182	0,297
s=80	2,815	0,016	0,074	0,079
s=90	1,704	0,005	0,019	0,031
s=100	1,031	$s^* = 86$;0,001	$s^* = 97$;0,007	0,011
s=110	0,624			$s^* = 104$;
s=120	0,378			
s=130	0,229			

s=140	0,138			
s=150	0,084			
s=160	0,051			
s=170	0,031			
s=180	0,019			
s=190	0,011			
$s^*=194$	0,006			

Literature

1. Джакупов К.Б.. О циклической оптимизации итерационных процессов//Изв.АН КазССР, сер. физ.-мат. н. 1980, №5.
2. Красносельский М.А., Крейн С.Г.. Итеративный процесс с минимальными невязкими // Математический сборник, нов. серия 31(73), вып. 2, 1952, 315–334.

Chapter 12. ABOUT APPLYING OF SCHEME OF PREDIKTOR-CORRECTOR IN HYDRODINAMICS

In algorithms of alternative directions (as type *Douglas-Rackford, Pisman-Rackford, Marchuk-Yanenko and others)* as opposed to explicit and implicit schemes, arise problem of raising of *boundary* conditions for intermediate grid functions. This problem is worsen for boundary conditions as type *fon Neiman*, and becomes insolvable in fields with curved boundary. With this point of view effective is applying of conditionally stable *explicit* schemes or absolutely stable *implicit* schemes.

For system of equations of hydrodynamics of viscous fluid, in view their irregularity, idea of applying of *implicit schemes* is realized in combination with semi-explicit schemes, technology of building of them is stated in **Chapters 9,10**.

Technology of scheme of predictor-corrector is demonstrated for starting-boundary task:

$$\rho(\frac{\partial v_e}{\partial t}+\sum_{m=1}^{3} v_m \frac{\partial v_e}{\partial x_m})+\frac{\partial p}{\partial x_e}=\rho F_e+$$

$$+\sum_{m=1}^{3}\frac{\partial}{\partial x_m}(\mu\frac{\partial v_e}{\partial x_m})-\frac{\partial}{\partial x_e}[(\frac{1}{3}\mu-\mu')\sum_{m=1}^{3}\frac{\partial v_m}{\partial x_m}],\ \ e=1,2,3,$$

$$\rho c_v(\frac{\partial T}{\partial t}+\sum_{m=1}^{3}v_m\frac{\partial T}{\partial x_m})=\sum_{m=1}^{3}\frac{\partial}{\partial x_m}(\lambda\frac{\partial T}{\partial x_m})+ \qquad (1)$$

$$+\mu\sum_{e=1}^{3}\sum_{m=1}^{3}(\frac{\partial v_e}{\partial x_m})^2-p\sum_{e=1}^{3}\frac{\partial v_e}{\partial x_e}-(\frac{1}{3}\mu-\mu')(\sum_{e=1}^{3}\frac{\partial v_e}{\partial x_e})^2,$$

$$\frac{\partial\rho}{\partial t}+\sum_{e=1}^{3}\frac{\partial\rho v_e}{\partial x_e}=0,\ p=R\rho T,\mu=\mu(T),\lambda=\lambda(T),$$

$$v_e\Big|_{t=0}=d_e,v_e\Big|_S=\varphi_e,e=1,2,3,T\Big|_{t=0}=d_T,T\Big|_S=\varphi_T$$

Predictor is monotonous scheme (see **Chapter 6,9,10**):

$$\rho_{ijk}^{n}v_{eijk}^{n+1}=[-p_{\tilde{x}_e}^{n+1}-Q_{eijk}^{n}]\tau_{n+1},\ \ e=1,2,3,$$

$$Q_{eijk}^{n}=\rho_{ijk}^{n}\Big\{-\tau_{n+1}^{-1}v_{eijk}^{n}+\sum_{m=1}^{3}[\frac{|v_{mijk}^{n}|+v_{mijk}^{n}}{2}v_{e\vec{x}_m}^{n}+\frac{|v_{mijk}^{n}|-v_{mijk}^{n}}{2}v_{ex_m}^{n}]-F_e\Big\}$$

$$-\sum_{m=1}^{3}(\mu^n v_{ex_m}^{n})_{\dot{x}_m}-\sum_{m=1}^{3}(\mu_m^n-\mu_{\min})v_{ex_m\dot{x}_m}^{n}+[(\frac{\mu^n}{3}-\mu'^n)\sum_{m=1}^{3}v_{m\tilde{x}_m}^{n}]_{\tilde{x}_e}, \qquad (2)$$

$$\rho_{ijk}^{n}c_v[(T_{ijk}^{n+1}-T_{ijk}^{n})/\tau_{j+1}+\sum_{m=1}^{3}(\frac{|v_{mijk}^{n}|+v_{mijk}^{n}}{2}T_{\vec{x}_m}^{n}+\frac{|v_{mijk}^{n}|-v_{mijk}^{n}}{2}T_{x_m}^{n})]=$$

$$=\sum_{m=1}^{3}(\lambda^n T_{x_m}^{n})_{\dot{x}_m}+\sum_{m=1}^{3}(\lambda_m^n-\lambda_{\min})T_{x_m\dot{x}_m}^{n}+$$

$$+\mu_{ijk}^{n}\sum_{e=1}^{3}\sum_{m=1}^{3}(v_{e\tilde{x}_m}^{n})^2-p_{ijk}^{n}\sum_{e=1}^{3}v_{e\tilde{x}_e}^{n}-(\frac{1}{3}\mu_{ijk}^{n}-\mu'^{n}_{ijk})(\sum_{e=1}^{3}v_{e\tilde{x}_e}^{n})^2,$$

$$\frac{\rho_{ijk}^{n+1}-\rho_{ijk}^{n}}{\tau_{n+1}}+\sum_{e=1}^{3}(\rho^n v^{n+1})_{e\tilde{x}_e}=0,$$

$$1\le i\le N_1-1,1\le j\le N_2-1,1\le k\le N_3-1,$$

$$\frac{\rho_{ijk}^{n+1}-\rho_{ijk}^{n}}{\tau_{n+1}}+(\rho^n v^{n+1})_{ex_e}+\sum_{\substack{m=1\\m\neq e}}^{3}(\rho^n\varphi^{n+1})_{m\tilde{x}_m}=0, i_e=0, 1\le i_q\le N_m-1,$$

$$\frac{\rho_{ijk}^{n+1}-\rho_{ijk}^{n}}{\tau_{n+1}}+(\rho^n v^{n+1})_{e\bar{x}_e}+\sum_{\substack{m=1\\m\neq e}}^{3}(\rho^n\varphi^{n+1})_{m\tilde{x}_m}=0, i_e=N_l, 1\le i_q\le N_m-1,$$

$$e=1,2,3, q=1,2,3, e\neq q,\ p^{n+1}=R\rho^{n+1}T^{n+1},$$

$$v_e^{n+1}\Big|_{S_h}=\varphi_e^{n+1}, e=1,2,3, T^{n+1}\Big|_{S_h}=\varphi_T^{n+1},$$

$$\mu_1^n=\mu_x^{vw}, \mu_2^n=\mu_y^{wu}, \mu_3^n=\mu_z^{uv}, \lambda_1^n=\lambda_x^{vw}, \lambda_2^n=\lambda_y^{wu}, \lambda_3^n=\lambda_z^{uv},$$

$$i_1=i, i_2=j, i_3=k.$$

Realization of so schemes is made by global iterations, that let in **Chapter 9** and **10**. In result calculated fields of components of velocity vector and pressure $v_e^{n+1}, e=1,2,3,\ p^{n+1}$ on time moment t_{n+1}.

Corrector is the same monotonous, but implicit scheme on time layout t_{n+2}. Corrector can consist from two types.

Lineage corrector. It takes type:

$$\rho_{ijk}^{n+1}v_{eijk}^{n+2}=\rho_{ijk}^{n+1}v_{eijk}^{n+1}-[p_{\tilde{x}_e}^{n+1}+G_{eijk}^{n+2}]\tau_{n+2},\ e=1,2,3, \quad (3)$$

$$G_{eijk}^{n+2}=\rho_{ijk}^{n+1}\left\{\sum_{m=1}^{3}[\frac{|v_{mijk}^{n+1}|+v_{mijk}^{n+1}}{2}v_{e\vec{x}_m}^{n+2}+\frac{|v_{mijk}^{n+1}|-v_{mijk}^{n+1}}{2}v_{ex_m}^{n+2}]-F_e\right\}$$

$$-\sum_{m=1}^{3}(\mu^{n+1}v_{ex_m}^{n+2})_{\dot{x}_m}-\sum_{m=1}^{3}(\mu_m^{n+1}-\mu_{\min})v_{ex_m\dot{x}_m}^{n+2}+[(\frac{\mu^{n+1}}{3}-\mu'^{n+1})\sum_{m=1}^{3}v_{m\tilde{x}_m}^{n+1}]_{\tilde{x}_e},$$

$$1\le i\le N_1-1, 1\le j\le N_2-1, 1\le k\le N_3-1,$$

$$v_e^{n+2}\Big|_{S_h}=\varphi_e^{n+2}, e=1,2,3$$

In this scheme well-known grid magnitudes are ρ^{n+1}, p^{n+1}, T^{n+1}, $\mu^{n+1} = \mu(T^{n+1})$ and components of speed $v_e^{n+1}, e = 1,2,3$, they are found by implicit scheme (2). In (3) difference derivatives taken on moment of time t_{n+2}, but coefficients by them on layout are t_{n+1}. In this case equations (3) are system of lineage algebraic equations, concerning components of speed

$$v_{eijk}^{n+2}, i = 1,\ldots,N_1 - 1, j = 1,\ldots,N_2 - 1, k = 1,\ldots,N_3 - 1$$

on time's layout t_{n+2}. Though system (3) has *diagonal prevalence* because of presence term of type $\dfrac{\rho^{n+1} v_{eijk}^{n+2}}{\tau_{n+2}}$, that is executes condition of convergence of iteration of Jakobi method.

Present (3) in compact type

$$\frac{\rho_{ijk}^{n+1}(v_{eijk}^{n+2} - v_{eijk}^{n+1})}{\tau_{n+2}} + p_{\tilde{x}_e}^{n+1} + G_{eijk}^{n+2} = 0, e = 1,2,3, \qquad (4)$$

$$1 \le i \le N_1 - 1, 1 \le j \le N_2 - 1, 1 \le k \le N_3 - 1,$$

$$v_e^{n+2}\Big|_{S_h} = \varphi_e^{n+2}, e = 1,2,3$$

Well, s- iteration indicated $v_{eijk}^{n+2,s}$, in limit for convergence iteration process of Jakobi $\lim\limits_{s\to\infty} v_{eijk}^{n+2,s} = v_{eijk}^{n+1}$. *Starting field of iteration by s = 0 equals* $v_e^{n+2,0} = v_e^{n+1}, e = 1,2,3.\backslash$

Calculate residuals s – iteration $R_e^s, e = 1,2,3$:

$$R_{eijk}^s = \frac{\rho_{ijk}^{n+1}(v_{eijk}^{n+2,s} - v_{eijk}^{n+1})}{\tau_{n+2}} + p_{\tilde{x}_e}^{n+1} + G_{eijk}^{n+2,s},$$

$$1 \le i \le N_1 - 1, 1 \le j \le N_2 - 1, 1 \le k \le N_3 - 1, v_{eijk}^{n+2,s}\Big|_{S_h} = \varphi_{eijk}^{n+2},$$

$$R_{eijk}^{s}\Big|_{S_h} = v_{eijk}^{n+2,s}\Big|_{S_h} - \varphi_{eijk}^{n+2} = 0, e = 1,2,3, s = 0,1,\ldots, s^*$$

Extract coefficients, state by v_{eijk}^{n+2} in G_{eijk}^{n+2}:

$$c_{+1\mu}^{n+1} = \frac{(\mu_x^{vw} - \mu_{\min}) + (\mu_{i+1jk}^{n+1} + \mu_{ijk}^{n+1})/2}{h_{xi+1}\hbar_{xi}} + \rho\frac{|u_{ijk}^{n+1}| - u_{ijk}^{n+1}}{2h_{xi+1}},$$

$$c_{-1\mu}^{n+1} = \frac{(\mu_x^{vw} - \mu_{\min}) + (\mu_{ijk}^{n+1} + \mu_{i-1jk}^{n+1})/2}{h_{xi}\hbar_{xi}} + \rho\frac{|u_{ijk}^{n+1}| + u_{ijk}^{n+1}}{2h_{xi}},$$

$$c_{+2\mu}^{n+1} = \frac{(\mu_y^{wu} - \mu_{\min}) + (\mu_{ij+1k}^{n+1} + \mu_{ijk}^{n+1})/2}{h_{yj+1}\hbar_{yj}} + \rho\frac{|v_{ijk}^{n+1}| - v_{ijk}^{n+1}}{2h_{yj+1}},$$

$$c_{-2\mu}^{n+1} = \frac{(\mu_y^{wu} - \mu_{\min}) + (\mu_{ijk}^{n+1} + \mu_{ij-1k}^{n+1})/2}{h_{yj}\hbar_{yj}} + \rho\frac{|v_{ijk}^{n+1}| + v_{ijk}^{n+1}}{2h_{yj}},$$

$$c_{+3\mu}^{n+1} = \frac{(\mu_z^{uv} - \mu_{\min}) + (\mu_{ijk+1}^{n+1} + \mu_{ijk}^{n+1})/2}{h_{zk+1}\hbar_{zk}} + \rho\frac{|w_{ijk}^{n+1}| - w_{ijk}^{n+1}}{2h_{zk+1}},$$

$$c_{-3\mu}^{n+1} = \frac{(\mu_z^{uv} - \mu_{\min}) + (\mu_{ijk}^{n+1} + \mu_{ijk-1}^{n+1})/2}{h_{zk}\hbar_{zk}} + \rho\frac{|w_{ijk}^{n+1}| + w_{ijk}^{n+1}}{2h_{zk}}$$

In Jakobi method *s + 1* – iteration $v_e^{n+2,s+1}, e = 1,2,3$ is identified by algorithm:

$$v_{eijk}^{n+2,s+1} = v_{eijk}^{n+2,s} - R_{eijk}^{s} / [\frac{\rho_{ijk}^{n+1}}{\tau_{n+2}} + \sum_{m=1}^{3}(c_{+m\mu}^{n+1} + c_{-m\mu}^{n+1})],$$

$$i = 1,\ldots,N_1 - 1,\ j = 1,\ldots,N_2 - 1,\ k = 1,\ldots,N_3 - 1,$$

$$v_{eijk}^{n+2,s+1}\Big|_{S_h} = \varphi_{eijk}^{n+2}, e = 1,2,3, s = 0,1,\ldots,s^*$$

Iterations are stopped by execution of criterion

$$\max_{\substack{1\le i\le N_1-1\\ 1\le j\le N_2-1\\ 1\le k\le N_3-1}} |R_{eijk}^{s^*}| \le \varepsilon, \quad \varepsilon \approx 0, \varepsilon \ne 0, e = 1,2,3$$

Interchange is made by this way: on layout of time t_{n+1} realized predictor, on layout t_{n+2} corrector. Of course, possible apply other iteration methods. For example, comfortably to write method *Libman-Zeidel* with help next difference operations: $\Lambda_m^{n+1} = c_{+m}^{n+1}\Delta_{+m} - c_{-m}^{n+1}\Delta_{-m}, m = 1,2,3$, where $\Delta_{+m} = T_{+m} - E, m = 1,2,3$ - operator of formation of differences forward, $\Delta_{-m} = E - T_{-m}, m = 1,2,3$ - operator of formation of differences backward, E – unit operator, $T_{+m}, m = 1,2,3$- operators of shear forward, $T_{-m}, m = 1,2,3$ - operators of shear backward.

System (4) is written in operator type

$$\frac{\rho^{n+1}(v_e^{n+2} - v_e^{n+1})}{\tau_{n+2}} = \sum_{m=1}^{3}(c_{+m\mu}^{n+1}\Delta_{+m} - c_{-m\mu}^{n+1}\Delta_{-m})v_e^{n+2} - p_{x_e}^{n+1} + \rho^{n+1}F_e, e = 1,2,3,$$

for which method of *Libman-Zeidel* is realized as

$$\frac{\rho^{n+1}\left(v_e^{n+2,s+1} - v_e^{n+1}\right)}{\tau_{n+2}} = \sum_{m=1}^{3}[c_{+m\mu}^{n+1}\left(T_{+m}v_e^{n+2,s} - v_e^{n+2,s+1}\right) - c_{-m\mu}^{n+1}\Delta_{-m}v_e^{n+2,s+1}] - p_{x_e}^{n+1} + \rho^{n+1}F_e,$$

$i = 1,\dots,N_1 - 1, j = 1,\dots,N_2 - 1, k = 1,\dots,N_3 - 1$,

$$v_e^{n+2,s+1}\Big|_{S_h} = \varphi_e^{n+2}, e = 1,2,3, s = 0,1,\dots,s^*$$

Similar way can use method of upper relaxation and other methods. Particularly, considerable effect gives applying of universal method of acceleration of convergence iterations **Chapter 11**.

Temperature T^{n+2} on moment of time t_{n+2} also calculate by method of *Jakobi* (or other method) as decision of next system of lineage algebraic equations with diagonal prevalence:

$$\rho_{ijk}^{n+1}c_v[(T_{ijk}^{n+2} - T_{ijk}^{n+1})/\tau_{n+2} + \sum_{m=1}^{3}(\frac{|v_{mijk}^{n+2}| + v_{mijk}^{n+2}}{2}T_{\bar{x}_m}^{n+2} + \frac{|v_{mijk}^{n+2}| - v_{mijk}^{n+2}}{2}T_{x_m}^{n+2})] =$$

$$= \sum_{m=1}^{3} (\lambda^{n+1} T_{x_m}^{n+2})_{\dot{x}_m} + \sum_{m=1}^{3} (\lambda_m^{n+1} - \lambda_{\min}) T_{x_m \dot{x}_m}^{n+2} +$$

$$+ \mu_{ijk}^{n+1} \sum_{e=1}^{3} \sum_{m=1}^{3} (v_{e\tilde{x}_m}^{n+2})^2 - p_{ijk}^{n+1} \sum_{e=1}^{3} v_{e\tilde{x}_e}^{n+2} - (\frac{1}{3} \mu_{ijk}^{n+1} - \mu'^{n+1}_{ijk})(\sum_{e=1}^{3} v_{e\tilde{x}_e}^{n+2})^2$$

Calculate residual s – iteration R_T^s :

$$R_{Tijk}^{s} = \rho_{ijk}^{n+1} c_v [(T_{ijk}^{n+2,s} - T_{ijk}^{n+1}) / \tau_{n+2} +$$

$$+ \sum_{m=1}^{3} (\frac{|v_{mijk}^{n+2}| + v_{mijk}^{n+2}}{2} T_{\bar{x}_m}^{n+2,s} + \frac{|v_{mijk}^{n+2}| - v_{mijk}^{n+2}}{2} T_{x_m}^{n+2,s})] -$$

$$- \sum_{m=1}^{3} (\lambda^{n+1} T_{x_m}^{n+2,s})_{\dot{x}_m} + \sum_{m=1}^{3} (\lambda_m^{n+2} - \lambda_{\min}) T_{x_m \dot{x}_m}^{n+2,s} - \mu_{ijk}^{n+1} \sum_{e=1}^{3} \sum_{m=1}^{3} (v_{e\tilde{x}_m}^{n+2})^2 +$$

$$- \mu_{ijk}^{n+1} \sum_{e=1}^{3} \sum_{m=1}^{3} (v_{e\tilde{x}_m}^{n+2})^2 + p_{ijk}^{n+1} \sum_{e=1}^{3} v_{e\tilde{x}_e}^{n+2} + (\frac{1}{3} \mu_{ijk}^{n+1} - \mu'^{n+1}_{ijk})(\sum_{e=1}^{3} v_{e\tilde{x}_e}^{n+2})^2$$

In method of *Jakobi* $s + 1$ – iteration $T^{n+2,s+1}$ is determined by algorithm:

$$T_{ijk}^{n+2,s+1} = T_{ijk}^{n+2,s} - R_{eijk}^{s} / [\frac{\rho_{ijk}^{n+1} c_v}{\tau_{n+2}} + \sum_{m=1}^{3} (c_{+m\lambda}^{n+2} + c_{-m\lambda}^{n+2})],$$

$$i = 1,\dots,N_1 - 1,\ j = 1,\dots,N_2 - 1,\ k = 1,\dots,N_3 - 1,$$

$$T_{ijk}^{n+2,s+1}\Big|_{S_h} = \varphi_{Tijk}^{n+2},\ s = 0,1,\dots,s^*$$

There are coefficients $c_{+m\lambda}^{n+2}, c_{-m\lambda}^{n+2}$ are similar by coefficients $c_{+m\mu}^{n+2}, c_{-m\mu}^{n+2}$. In lineage corrector in $c_{+m\lambda}^{n+2}, c_{-m\lambda}^{n+2}$ used coefficient of thermal conduction $\lambda^{n+1} = \lambda(T^{n+1})$ on layout t_{n+1} .

Iterations are stopped by execution of criterion

$$\max_{\substack{1\le i\le N_1-1\\ 1\le j\le N_2-1\\ 1\le k\le N_3-1}} | R_{Tijk}^{s^*} | \le \varepsilon, \quad \varepsilon \approx 0, \varepsilon \ne 0$$

Non-lineage corrector. It has type:

$$\rho^{n+1} v_e^{n+2} = \rho^{n+1} v_e^{n+1} - [p_{\tilde{x}_e}^{n+1} + \overline{G}_e^{n+2}] \tau_{n+2}, \ \ e = 1,2,3,$$

$$\overline{G}_e^{n+2} = \rho \left\{ \sum_{m=1}^{3} [\frac{| v_m^{n+2} | + v_m^{n+2}}{2} v_{e\vec{x}_m}^{n+2} + \frac{| v_m^{n+2} | - v_m^{n+2}}{2} v_{ex_m}^{n+2}] - F_e \right\}$$

$$- \sum_{m=1}^{3} (\mu^{n+1} v_{ex_m}^{n+2})_{\dot{x}_m} - \sum_{m=1}^{3} (\mu_m^{n+2} - \mu_{\min}) v_{ex_m \dot{x}_m}^{n+2} + [(\frac{\mu^{n+1}}{3} - \mu'^{n+1}) \sum_{m=1}^{3} v_{m\tilde{x}_m}^{n+1}]_{\tilde{x}_e},$$

$$1 \le i \le N_1 - 1, 1 \le j \le N_2 - 1, 1 \le k \le N_3 - 1,$$

$$v_e^{n+2} \Big|_{S_h} = \varphi_e^{n+2}, e = 1,2,3$$

In $\mu_m^{n+2}, m = 1,2,3$ used coefficient of viscosity μ^{n+1} and components of speed $v_e^{n+2}, e = 1,2,3$.

Execute residuals *s* – iteration $R_e^s, e = 1,2,3$:

$$R_{eijk}^s = \frac{\rho_{ijk}^{n+1} (v_{eijk}^{n+2,s} - v_{eijk}^{n+1})}{\tau_{n+2}} + p_{\tilde{x}_e}^{n+1} + \overline{G}_{eijk}^{n+2,s},$$

$$1 \le i \le N_1 - 1, 1 \le j \le N_2 - 1, 1 \le k \le N_3 - 1,$$

$$v_{eijk}^{n+2,s} \Big|_{S_h} = \varphi_{eijk}^{n+2},,$$

$$R_{eijk}^s \Big|_{S_h} = v_{eijk}^{n+2,s} \Big|_{S_h} - \varphi_{eijk}^{n+2} = 0, e = 1,2,3, s = 0,1,\dots, s^*$$

In *non-lineage* corrector *s + 1* – iteration $v_e^{n+2,s+1}, e = 1,2,3$ is determined by similar algorithm of *Jakobi*:

$$v_{eijk}^{n+2,s+1} = v_{eijk}^{n+2,s} - R_{eijk}^{s} / [\frac{\rho_{ijk}^{n+1}}{\tau_{n+2}} + \sum_{m=1}^{3} (c_{+m\mu}^{n+2,s} + c_{-m\mu}^{n+2,s})],$$

$$i = 1,\ldots,N_1 - 1,\ j = 1,\ldots,N_2 - 1, k = 1,\ldots,N_3 - 1,$$

$$v_{eijk}^{n+2,s+1}\Big|_{S_h} = \varphi_{eijk}^{n+2}, e = 1,2,3, s = 0,1,\ldots,s^*$$

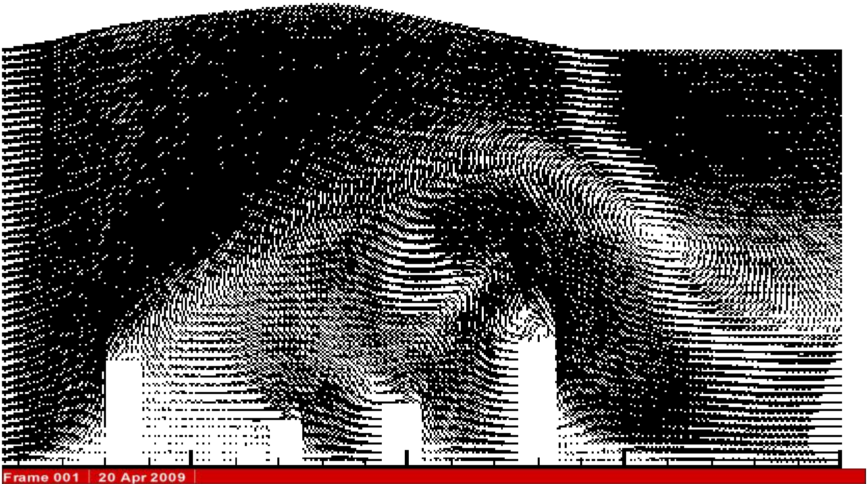
Iterations are stopped by execution of criterion

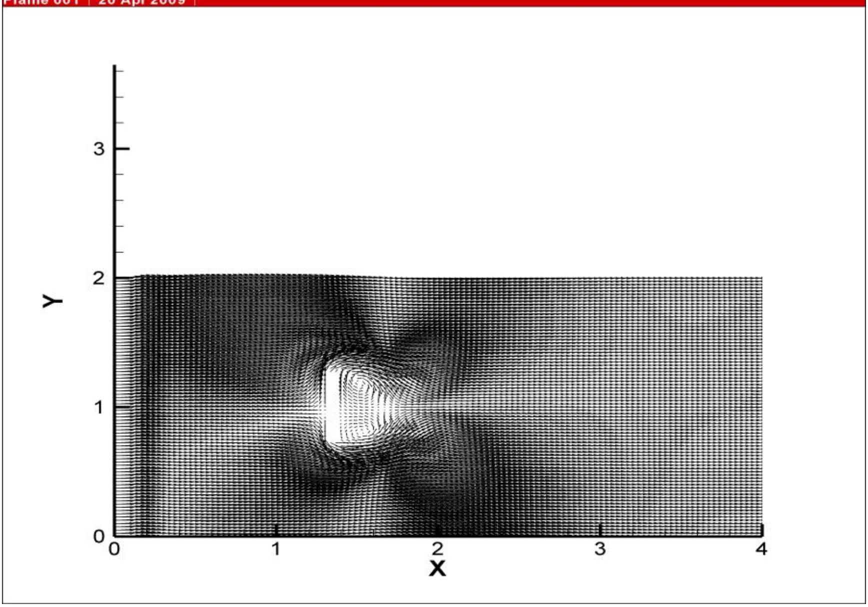
$$\max_{\substack{1\le i\le N_1-1 \\ 1\le j\le N_2-1 \\ 1\le k\le N_3-1}} | R_{eijk}^{s^*} | \le \varepsilon, \quad \varepsilon \approx 0, \varepsilon \ne 0, e = 1,2,3$$

Algorithm of calculation of temperature T^{n+2} on moment of time t_{n+2} is realized after definition $v_e^{n+2}, e = 1,2,3$ and indicated above

Literature

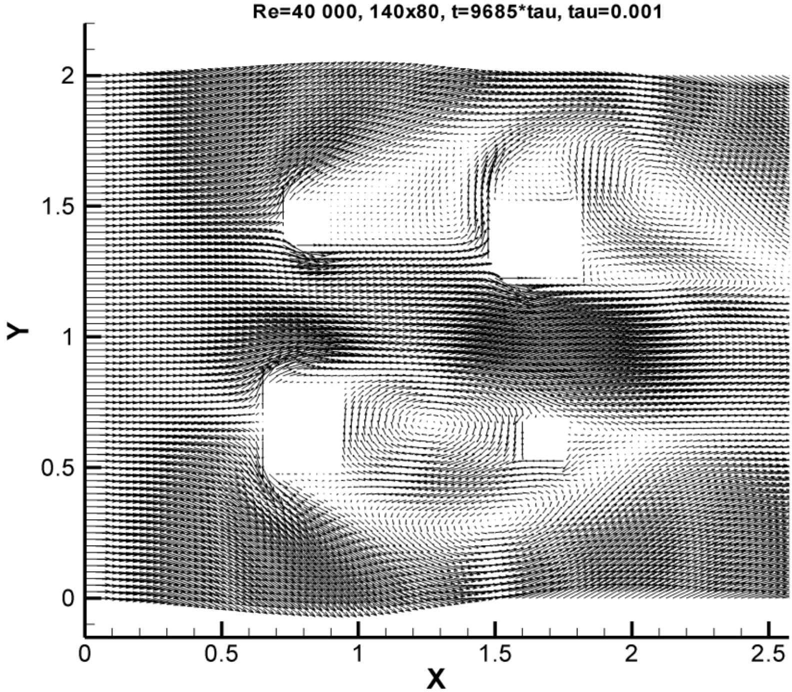
1. Демидович Б.П. и Марон И.А. Основы вычислительной математики. - изд-во «Наука», М.,1970, с.661.
2. Яненко Н.Н. Метод дробных шагов решения многомерных уравнений математической физики.- Сиб. отд. «Наука», Новосибирск, 1967, с.195.
3. Джакупов К.Б., Гончаров А.И. Численное моделирование диффузионного горения в закрученных потоках двухфакельной топки // Сибирский физико-технический журнал, 1991, вып.6/91,с.83-93.





The upper figure shows vortex separation in flow around buildings with different height. Uniform stream runs from the left side. Grid 140x80.

The lower drawing presents a picture of plate cross flow. Developed behind the obstacle two vortexes develop with time into interacting with each other and flow-drifted nonsymmetric vortexes.



Field of actual velocity vector $\vec{v} = \vec{\bar{v}} + \vec{v}'$ *in turbulent flow around four buildings, top view. Dimensionless averaging time is equal Dg=2*τ *,* τ *=**tau.***

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